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FRP-Confined concrete analytical axial stress model evaluation

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Abstract

Due to the necessity to strengthen existing reinforced concrete structures, innovative materials such as Fiber Reinforced Polymers (FRP's) may be used to comply with increasing loads and modern regulations. Externally bonded FRP overlays have been used to increase the flexural, shear and compressive capacity of concrete. The use of externally bonded FRP's to confine circular concrete structural elements has shown experimentally a significant increase of the compressive concrete strength. In the course of the development of composite materials since the 1990s, along with the methodologies concerning their application, a plethora of models have also emerged in order to determine the FRP-confined compressive concrete stress with good accuracy. This paper presents a study on the compressive strength of concrete confined with externally bonded fiber reinforced composite materials. In order to evaluate the experimental data and analytical models an extensive database of experimental data is created from experiments reported in the literature, sharing common test features (conventional concrete cylindrical specimens confined using FRP composites of various fibers aligned perpendicular to the longitudinal axis of the cylinder). Based on the collected experimental data a statistical analysis is performed in order to identify the major parameters that affect the ultimate confined axial stress. Furthermore, the experimental data are used in a series of available models in order to determine the analytical predictions of ultimate stresses in comparison to the experimental ones. Statistical methods are used to quantify the analytical models' efficiency and to identify major factors that affect the models' performance.

Key words: FRP, Confinement, Concrete, Stress, Statistical Models

1 Introduction

Older concrete structures can not cope with the increasing demands of modern-day requirements. Today it is more imperative than ever to find solutions and develop methods that can help rehabilitate or strengthen existing structures. A technology that thrusted into the forefront in the dawn of the new millennium is the use of Fiber Reinforced Polymers (FRP's) as an externally bonded strengthening material. More specifically, the use of composite materials consisting of strong fibers impregnated in a polymer matrix can help strengthen existing structures and achieve performance required by modern codes. FRPs can be used as external reinforcement to increase flexural or shear capacity of structural elements. However, they can be even more effective when used for the confinement of concrete structural elements.

Concrete confinement is known to increase concrete compressive strength, which is the most important parameter for concrete design. Numerous studies have examined the performance of such confined concrete cylindrical specimens. The most important finding was that confined concrete members exhibited a significantly higher ductility, a much-desired structural property in earthquake prone regions. Fig. 1 shows the development of the confining stresses in a circular concrete specimen.

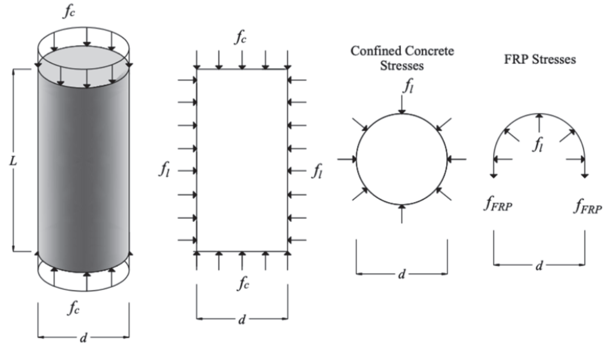

Figure 1. Scheme of stresses due to FRP confining action in circular concrete element [1]

During the axial strain the circular concrete element is expanded transversely. At this point the composite material is activated as transverse tensile deformations are developed in its fibers. At the same time the composite material exerts transverse compressive confinement stress vertically to the axis of the element [2].

In a circular specimen the transverse strength of the confinement is equally forced to the concrete in the opposite direction. The strength may be calculated using Eq (1):

$$\sigma_3 = \frac{2 \cdot t_{FRP} \cdot \sigma_{FRP}}{D} = \frac{2 \cdot t_{FRP} \cdot E_{FRP} \cdot \varepsilon_{FRP}}{D} \quad (1)$$

σ_3 - The transverse confinement strength (MPa)
σ_c - Strength of the axial compressive load (MPa)

σ_{FRP} - The shear strength of the FRP (MPa)
E_{FRP} - The FRP elasticity modulus (GPa)
ε_{FRP} - The FRP strain
t_{FRP} - The FRP thickness (m)
D - The diameter of the specimen (including the thickness of the FRP) (m)

The confinement aims in the prevention of concrete cracking by increasing the strength and the strain of the element until the rupture of the FRP [3].
This paper analyzes the equations of various models developed during the last 40 years by scholars, trying to determine the extra strength given to a cylindrical unarmed concrete specimen under uniaxial compression according to the composite material, the matrix, the number of layers and the orientation of fibers, the geometry of the specimen and the quality of the concrete used.

2 Experimental database

This study focuses on the collection and evaluation of data deriving from a series of experiments on confined circular concrete structural elements. There is a number of parameters affecting the measurements and the subsequent processing of test results, such as:

- Laboratory conditions (temperature, humidity etc.)
- The equipment used (hydraulic test machines, loading rate, sensor sensitivity etc.)
- The specimen and sensor's installation (strain gauge, LVDT placement etc.)
- Accuracy of measurements (noise and data processing)

There are also important parameters related to the fabrication of the specimens, which could have a significant impact on the results of the experimental process such as:

- Suitability of materials and detailed construction of the concrete specimens (water/cement ratio, cement quality, adequate curing conditions etc.)
- Appropriate preparation of the specimen (careful cleaning, application of matrix etc.)
- Application of composite materials (type of composite material, selection of weaving, placement and orientation of fibers, number of layers, quantity and density of resinous materials, proper confinement, length of coating etc.)

In the context of this paper, all available experimental test results collected concern cylindrical concrete specimens with compressive strengths of 12 to 58 MPa, confined by composite materials. Concerning the composite materials, there are several categories with different properties. In addition to the mechanical characteristics (Young's modulus, maximum tensile strength, etc.) other parameters play important role such as the application of the materials and the preparation of the concrete. However, in the context of this study we only focused on the fiber type. Therefore, the composite material categories examined in this paper are the following:

- Carbon Fiber Reinforced Polymer: CFRP
- Glass Fiber Reinforced Polymer: GFRP
- Aramid Fiber Reinforced Polymer: AFRP
- High – Ultra High Modulus – CFRP: UM_UHM
- Unbonded Tube (CFRP, GFRP, AFRP): UB_TUBE

The first three materials are the most widely used in experiments found in the literature. Due to the different mechanical properties of each material, they exhibit different behavior during the compression of the confined concrete specimen. The CFRP with 1 layer of coating into concrete C30 provides strength of 35-74 MPa. Having the same parameters, GFRP produces a ultimate compressive strength of 30-60 MPa, whereas AFRP 40-50 MPa. A further distinction was made, creating the category of UM_UHM which referrers to CFRP materials with a high or ultra-high Young's modulus (E>300 GPa) and UB_TUBE which covers carbon, glass or aramid fibers used in TUBE form confinement. Therefore, due to the type of the fibers used as confining material, the ultimate stress varies significantly.

The experimental database developed in this study consists of 1040 specimens under compressive stress uniaxially progressed at a continuous rate until the failure of the unconfined concrete element is reached or until the rupture or detachment of the confinement fibers. The characteristics of the specimens are displayed in Table 1.

Table 1. Number of tested specimens for each composite category

Composite Category	**No of specimens**
CFRP	573
GFRP	234
AFRP	67
UM_UHM	50
UB_TUBE	116
NORMAL_ALL	**1040**

The above classification becomes clearer when expressed in percentages, as shown in Fig. 2. Regarding the composite material the CFRP dominate the experimental database, covering 55% of the data, followed by GFRP with 23%, Tubular Fibers with 11%, AFRP with 6% and finally UHM with 5 %.

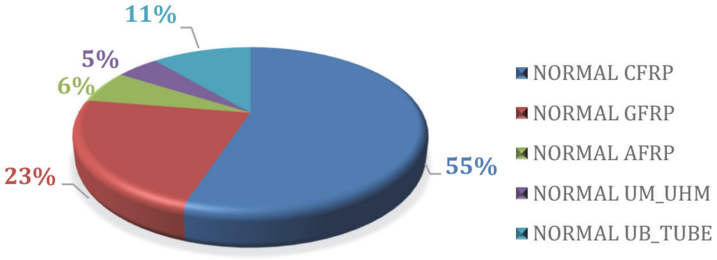

Figure 2. Percentage distribution of specimens per type of fiber and concrete strength

3. Analytical compressive stress models

There are numerous models developed over the last 40 years targeting to calculate the additional strength that a concrete element will exhibit when confined by composite materials. The models are based on several parameters related to the material mechanical properties and geometry. The challenge in this study was to evaluate some commonly used models using data collected in the previously described experimental database. Using the 1040 tested specimens it is expected to identify the most important factors in each model and evaluate its effectiveness. The analytical models are detailed in Table 2.

Table 2. Examined stress models

YEAR	AUTHOR	REFERENCE	STRESS MODEL
1996	Mirmiran	[4]	$f'_{cc} = f'_{co} + 4.269 \times (f_{lu})^{0,587}$
1999	Spoelstra & Monti	[5]	$\frac{f'_{cc}}{f'_{co}} = 0.2 + 3 \times \left(\frac{f_{lu}}{f'_{co}}\right)^{0,85}$
2002	Moran & Pantelides	[6]	$\frac{f'_{cc}}{f'_{co}} = 1 + k_1 \times \left(\frac{f_{lu,a}}{f'_{co}}\right)$ $k_1 = 4.14\ for\ bonded\ FRP\ shell$ $k_1 = 2.33\ for\ unbonded\ FRP\ shell$
1994	EC_2 (EN1992-1-1)	[7]	$f_{ck,c} = f_{ck} \times \left(1.0 + 5.0 \times \frac{\sigma_2}{f_{ck}}\right) --\rightarrow \sigma_2 \leq 0.05 f_{ck}$ $f_{ck,c} = f_{ck} \times \left(1.125 + 2.5 \times \frac{\sigma_2}{f_{ck}}\right) --\rightarrow \sigma_2 > 0.05 f_{ck}$ $\varepsilon_{fj} = \min(\varepsilon_{cu}\,; 0.015)$ $\rho_f E_f = \left(\frac{4 \times t_{frp}}{D}\right) \times E_{frp} \times 1000$ $\sigma_2 = f_{lu} = 0.5 \times \rho_f E_f \times \varepsilon_{fj}$
2020	Papakonstantinou	[1]	$\frac{f'_{cc}}{f'_{co}} = 1 + 2.1 \times \left(\frac{f_{lu}}{f'_{co}}\right)^{0,86}$

The main parameters identified in the experiments are:
$E_{frp} = E_l = E_f$ - Elastic modulus of composite material (GPa)
$f_{frp} = f_f$ - Ultimate strength of composite material (MPa)

$e_{frp} = e_f$ - Ultimate axial strain of composite material
$t_f = t$ - Total thickness of composite material (mm)
$D = d$ - Specimen's diameter (mm)
$H = h$ - Specimen's height (mm)
$f'_{co} = f_{ck}$ - Unconfined concrete strength (MPa) (Experimental value)
f'_{cu} - Confined concrete strength (MPa) (Experimental value)
$f'_{cc} = f_{ck,c}$ - Confined concrete strength (MPa) (Analytical value)
$f_{lu} = f_{lu,a} = p_3$ - Lateral strength of concrete walls vertical to the longitudinal axis of the specimen (MPa)
r_f - Percentage of FRP in the specimen $\left(\rho_f = \frac{4 \times t_f}{D}\right)$
e_{co} - Unconfined concrete strain

Examining the equations, it seems that the models of Spoelstra & Monti [5], Moran & Pantelides [6] and Papakonstantinou [1] have similar structure, since all three models employ lateral and unconfined concrete strength as their major parameters. However, the empirical equation suggested by Spoelstra & Monti [5] results in a totally incorrect unconfined concrete compressive strength ($f'_{cc}=0.2\,f'_{co}$).

4 Evaluation of the models

4.1 Diagrams

Having calculated the strength of reinforced concrete using the aforementioned equations, a statistical analysis is carried out in order to identify the efficiency of each model. The statistical analysis correlates the experimental value of the strength of confined concrete (to the one calculated through each model An ideal model should be able to predict every single experimental value. Therefore, the optimal graph is for every point to fall onto the diagonal line of 45°. Fig. 1 shows an indicative diagram illustrating the line of 45°, as well as those of 30° and 60°.

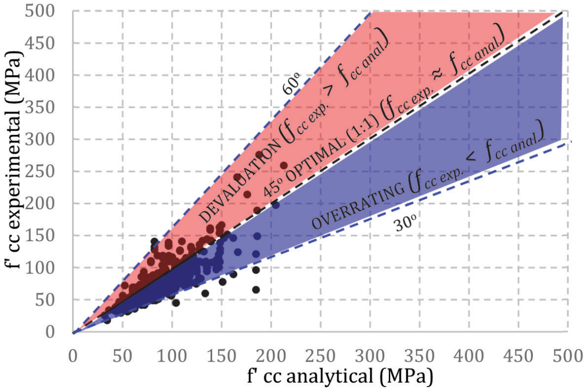

Figure 3. Indicative diagram of *f'_{cc} experimental vs f'_{cc} analytical*

As shown in Fig. 3, when all the values fall exactly on the 45° line there is a perfect correlation (relation 1:1) indicating a theoretically "ideal" model. When the values are between 45° and 30° the analytical value of stress is higher of the experimental one and therefore the model overestimates the durability. In case the values are between 45° and 60° then the model underestimates the strength of confined concrete since the analytical value is lower than the experimental one.

The diagrams in Fig. 4 present the results obtained from each examined model. It should be noted that all specimens used are cylindrical and of normal strength concrete, confined with carbon fibers. In particular, it is obvious that the graphs shown in Fig. 4(b, c, e) indicate that there is a good concordance between experimental and analytical values obtained through Spoelstra & Monti [5], Moran & Pantelides [6] and Papakonstantinou [1] models, whereas in Fig. 4(a) Mirmiran's model [4] underestimates the confinement strength and in Fig. 4(d) the analytical methodology suggested by Eurocode 2 overestimates the effect of confinement. However, the model of Spoelstra & Monti [5] results in more data above the 60° line than the other two well performing models. Moreover, from a comparison between the data points shown in Fig. (c) and (e) it is obvious that the model of Papakonstantinou [1] performs slightly better, having only two points under the 30° line.

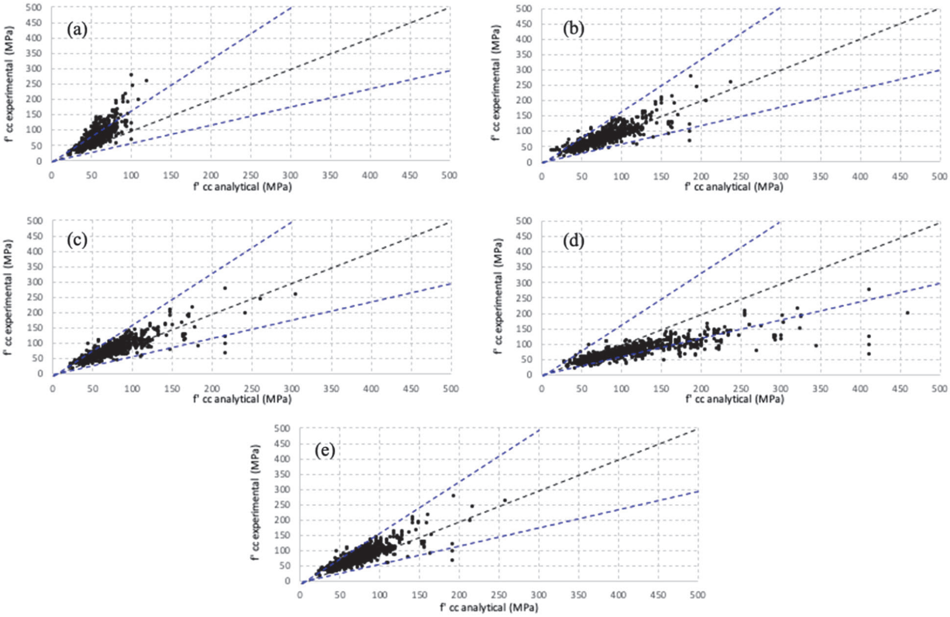

Figure 4. for CFRP-confined concrete calculated using (a) the Mirmiran model [4], (b) the Spoelstra & Monti model [5], (c) the Moran & Pantelides model [6], (d) the methodology suggested by EC_2 (EN 1992-1-1) [7] and (e) the Papakonstantinou model [1]

4.2 Statistical analysis

The overall evaluation of the models can be performed using the values of the ratios $f_{cc,exp}/f_{cc,ana}$. In a theoretically perfect model, the average of the ratios for a model should be equal to one, with a minimal standard deviation. If x_i is the experimental to analytical stress ratio for a particular specimen, the average and standard deviation, can be expressed by Equations (2) and (3), respectively.

$$\overline{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \ldots + x_n}{n} \tag{2}$$

$$S = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n-1}} \tag{3}$$

The application of Eq. (2) and Eq. (3) to the models is illustrated in Fig. (5). It seems that the model of Spoelstra & Monti [5] has the best average value, followed by the models suggested by Moran & Pantelides [6], and by Papakonstantinou [1], which have a quite small standard deviation. It is noted that all three models are empirical, but as mentioned earlier the model suggested by Spoelstra & Monti does not work in extreme cases (no or very small confinement).

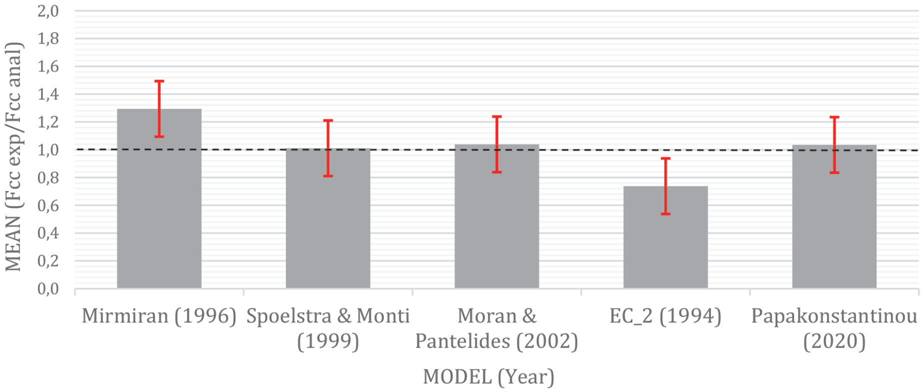

Figure 5. Mean value $f_{cc\ exp.}/f_{cc\ anal.}$ applied to all normal strength concrete models

The same results, regarding the efficiency of the models, are observed when Lin's coefficient is used. The models of Spoelstra & Monti (1999), Moran & Pantelides (2002) and Papakonstantinou (2020) remaining in the top rank. Lin's concordance coefficient is measuring the deviation from the 1:1 ratio of the compared values. Perfect concordance results in a value of 1 while the lower the values the higher the deviation from the 45° line. Lin's concordance coefficient is calculated by the following equations:

$$\rho_c = \frac{2 \times S_{xy}}{S_x^2 + S_y^2 + \left(\overline{x} - \overline{y}\right)^2} \tag{4}$$

$$\overline{x} = \frac{1}{N}\sum_{n=1}^{N} x_n \tag{5}$$

$$S_x^2 = \frac{1}{N}\sum_{n=1}^{N} \left(x_n - \overline{x}\right)^2 \tag{6}$$

$$S_y^2 = \frac{1}{N}\sum_{n=1}^{N} \left(y_n - \overline{y}\right)^2 \tag{7}$$

$$S_{xy} = \frac{1}{N}\sum_{n=1}^{N} \left(x_n - \overline{x}\right)\left(y_n - \overline{y}\right) \tag{8}$$

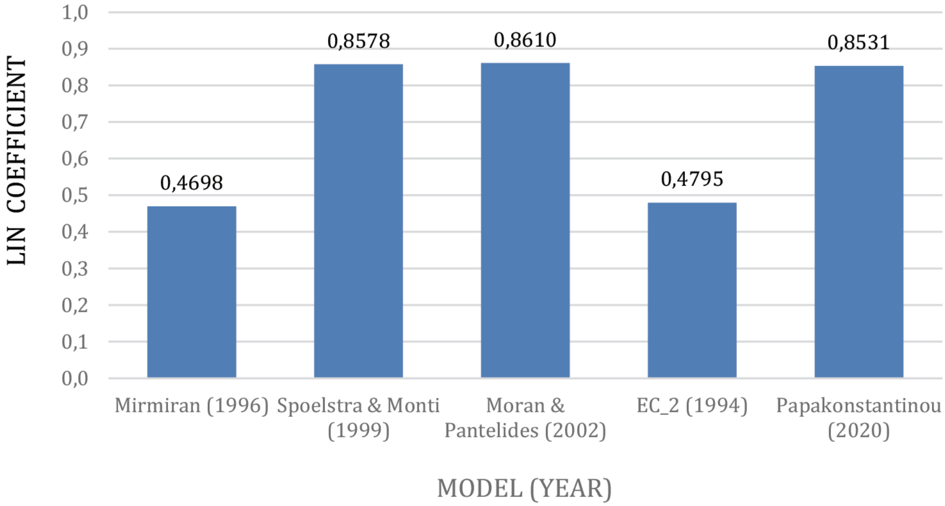

Figure 6. Lin's concordance coefficient for each examined model

5 Conclusion

In this study an experimental database containing experiments on FRP confined cylindrical concrete specimens was created. A series of analytical models that predict the ultimate concrete confined strength was examined using the experimental data and evaluated using a series of statistical methods. The following conclusions can be drawn:

- The most widely used confining material is the CFRP.
- The equations provided by Spoelstra & Monti [5], Moran & Pantelides [6] and Papakonstantinou [1], are quite similar. These three models are very efficient regardless the evaluation method used.
- There are models that permanently devaluate or overestimate the confinement stress, based on the very low values of the Lin coefficient. These models are obviously the least reliable.
- The models that exhibit the best behavior are those having a simple form, while more complex models using exponential forms or involving multiple parameters (E, f, D, t) exhibit worse behavior and the calculated confinement strength is significantly different from the experimental one.

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