1st Croatian Conference on Earthquake Engineering
1CroCEE
22-24 March 2021 Zagreb, Croatia

DOI: https://doi.org/10.5592/CO/1CroCEE.2021.87

FRP-confined concrete: A comparison analysis of ultimate axial strain models

Maria Valasaki[1], Christos Papakonstantinou[2]

[1] *PhD candidate*, Department of Civil Engineering, University of Thessaly, Volos, Greece

[2] *Assistant Professor*, Department of Civil Engineering, University of Thessaly, Volos, Greece, *cpapak@civ.uth.gr*

Abstract

Fiber Reinforced Polymers (FRP's) are widely used as composite materials in Civil Engineering applications in order to rehabilitate or strengthen existing structural elements. FRP's have been used as flexural and shear reinforcement and also as a confining material mostly for concrete columns. Several studies have identified a large increase in terms of strength and axial deformation for confined concrete specimens. This paper presents a comprehensive review and evaluation of analytical models developed to predict the maximum longitudinal deformation of confined circular concrete columns subjected to compression. The equations of the models are applied on an extensive experimental database regarding test results of 847 FRP-confined cylindrical concrete columns under uniaxial compression. The database contains sections of concrete of different strengths confined with different types of FRP's. The performance of each strain model is assessed through various statistical indicators, such as Root Mean Square Error (RMSE) and the Nash–Sutcliffe model efficiency coefficient (NSE). These statistical indexes compare the concordance between experimental maximum recorded strains and analytical predictions of ultimate strain. The most efficient models are identified, leading to important observations regarding the influence of main parameters on the behavior of FRP-confined concrete, such as the concrete and fiber material properties.

Key words: FRP, composites, confinement, concrete, strain, RMSE index, NSE index

1 Introduction

A relatively new field of activity in civil engineering is the research and application of methods in repairing and strengthening reinforced concrete structures that are damaged due to seismic action, age, environmental conditions or in need for rehabilitation due to change in use and therefore change in loads. Structural strengthening and rehabilitation can be achieved using externally bonded composite materials consisting of carbon, glass or aramid fibers impregnated in a typically organic matrix. These materials have significant advantages such as high strength, very low weight, extremely high durability, availability in long lengths, while they are not affected by corrosion [1].

When composite materials are used for concrete confinement, they are activated when concrete expands transversely. At this point, tensile deformations are developed in the fibers of the composite material leading to transverse compressive stress. The fibers continue to stress axially until they reach their maximum capacity, while offering active confinement to the concrete core. The transverse compressive stress the FRP offers leads to a significant increase of maximum compressive strength and axial deformations. In particular, if we consider a cylindrical concrete element, as shown in Fig. 1, having a diameter *d*, externally bonded by a composite material with thickness t_f, modulus of elasticity E_f, the lateral stress developed in the composite material σ_λ (equal and opposite strength is exerted on the concrete, as reinforcement stress), is provided by the equation:

$$\sigma_l = \frac{2\,t_f}{d}\sigma_f = \frac{2\,t_f}{d}E_f\,\varepsilon_f \tag{1}$$

where σ_f is the tensile stress and ε_f the stain respectively.

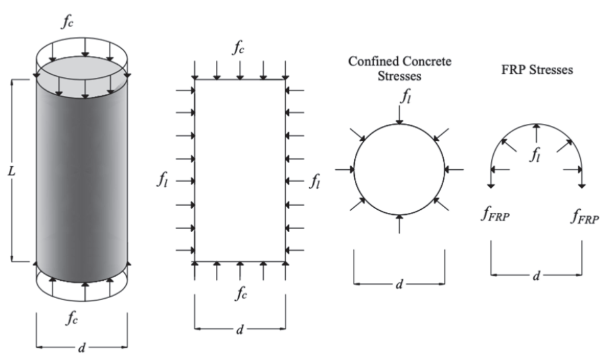

Figure 1. Cylindrical element under axial stress confined by composite materials and stress development due to active confinement [2]

The active confinement of the concrete element and thus the resulting confinement stress contribute to:

1. The prevention of excessive concrete cracking and therefore in the increase of durability.
2. The increase of the deformability of the concrete and thus its ductility.
3. The increase of the bond between rebars and concrete, providing a better force transfer mechanism prevents the premature slippage of the rebars.

In this context, this study focuses in the longitudinal deformation of circular confined concrete elements under axial compression and it is based on scientific research started in the early 1980's until now, recording models developed to quantify experimental applications for any type of fiber and specimen.

2 Experimental Database

Since the emergence of FRP composites as confining materials for reinforced concrete (RC) structures, a large number of experimental and analytical studies have been conducted in order to understand and model the behavior of RC elements confined by FRPs. Through an extensive literature a series of experiments on confined circular concrete specimens were identified. Data from these studies was collected such the geometrical characteristics of the specimens, mechanical properties of the concrete and the composite material. All reported parameters were congregated in an comprehensive experimental database.

The specimens examined in this study are of normal strength concrete with a circular cross-section, confined by composite materials with fibers parallel to the perimeter of the element. In total, as shown in Table 1, 847 specimens were used. The concrete of these specimens had a compressive strength of 12 to 56 MPa. The following categories were identified based on the type of composite material used for confinement:

- **CFRP**: carbon fibers
- **GFRP**: glass fibers
- **AFRP**: aramid fibers
- **HM_UHM_CFRP** (High Modulus – Ultra High Modulus CFRP): carbon fibers with a high or ultra high elasticity modulus (E > 350 GPa).
- **UB_TUBE** (Unbonded Tube / CFRP, GFRP, AFRP): carbon, glass or aramid fibers in the form of a prefabricated mantle (seamless) that can only be applied to new constructions.

This categorization becomes clearer when expressed in percentages, as shown in Fig. (2). Regarding the composite material the carbon fibers dominate, covering 50 % of the data, followed by glass fibers (25 %). The smaller percentage of the experimental data refer to tube fibers, aramid fibers and high elastic modulus carbon fibers with 12 %, 8

% and 5 % respectively. Although carbon fibers are significantly more expensive and do not necessarily perform better than glass fibers when used for concrete confinement. Table 1. Test database regarding the fiber of composite material and the strength of unconfined concrete.

3. Analytical compressive strain models

Thanks to a comprehensive literature review, numerous analytical models were identified. These models could be used to calculate the ultimate deformation of circular concrete FRP-confined specimens subjected to compressive loads. Due to their large number, it was decided to focus on the six most representative ones (see Table 2). The models shown in Table 2 vary in terms of simplicity and performance. Some of them consist of just one simple equation, while others are significantly more complicated. The parameters that are commonly used are the unconfined concrete strength (f'_{co}), and the nominal lateral stress during failure (f'_{lu}), which depends upon the confining material strength and thickness, as well as specimen diameter. The unconfined concrete strain is typically considered equal to 0.002 by most models, while Ozbakkaloglu and Lim [3] are the only ones who provide a specific equation for its calculation.

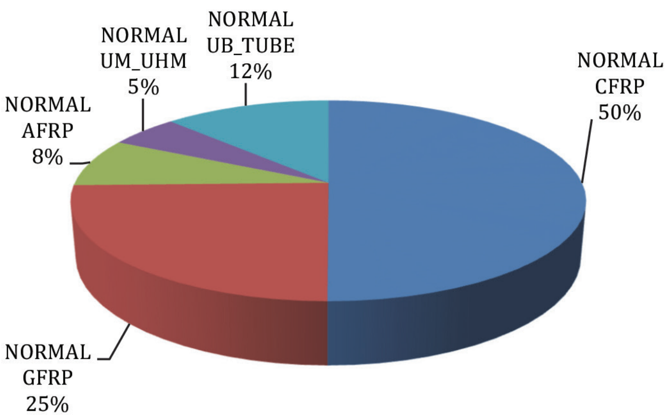

Figure 2. Percentage distribution of specimens per type of fiber.

The main equations used in these models were derived using empirical formulas based on limited number of experimental results. Aim of this paper is to validate these models using an extended database so as to prove their efficiency and determine the dominant factors in strain estimation. It is evident, that the values of f_{lu}, ε_{cu} and f_{co} are dominant in all models and are assumed to be the most important parameters in the estimation of the ultimate strain. In almost all models, the authors suggest using a value of 0.002 as the unconfined concrete maximum strain, ε_{co}, which in some cases deviates from measured experimental values, depending on concrete strength. It is obvious that the unconfined concrete strain can affect significantly the performance of the model. In that respect, Ozbakkaloglu and Lim [3], introduced an equation for the calculation of ε_{co}, which uses the concrete compressive strength as a variable. This model, being the most recent among those examined, was expected to perform better compared to the simpler and older models.

Table 2. Examined Ultimate Strain Models.

YEAR	AUTHOR	REFERENCE	STRAIN MODEL
1997	Karbhari & Gao	[4]	$\varepsilon_{cu} = \varepsilon_{co} + 0.01\left(\frac{f_{lu}}{f'_{co}}\right)$ $\varepsilon_{co} = 0.002$
1998	Samaan, Mirmiran & Shahaw	[5]	$\varepsilon_{cu} = \frac{f'_{cu} - f_o}{E_2}$, $f'_{cu} = f'_{co} + k_1 f_r$ $k_1 = 6.0\, f_r^{-0.3}\ [MPa]$, $f_r = \frac{2\, f_j\, t_j}{D}$ $f_o = 0.872\, f'_c + 0.371\, f_r + 6.258\ [MPa]$ $E_2 = 245.61\, f_c'^{0.2} + 1.3456\, \frac{E_j t_j}{D}\ [MPa]$
1999	Miyauchi et al	[6]	$\varepsilon_{cu} = \varepsilon_{co} + \varepsilon_{co}(15.87 - 0.093 f'_{co}) \times \left(\frac{f_{lu}}{f'_{co}}\right)^{(0.246+0.0064 f'_{co})}$
1999	Spoelstra & Monti	[7]	$\varepsilon_{cu} = \varepsilon_{co}\left[2 + 1.25\left(\frac{E_c}{f'_{co}}\right)\left(\frac{f_{lu}}{f'_{co}}\right)^{0.5} \varepsilon_{lu}\right]$ $\varepsilon_{co} = 0.002$
2002	Ilki, Kumbasar & Ko	[8]	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 15.156\left(\frac{f_{lu}}{f'_{co}}\right)^{0.735}$ $for\ CFRP\ confinement$
2013	Ozbakkaloglu & Lim	[3]	$\varepsilon_{cu} = c_2\, \varepsilon_{co} + k_2\left(\frac{K_l}{f'_{co}}\right)^{0.9} (\varepsilon_{h,rup})^{1.35}$ $k_2 = 0.27$, $c_2 = 2 - \frac{(f'_{co} - 20)}{100}$ $and\ c_2 \geq 1$ $\varepsilon_{co} = (-0.067 f_{co}'^2 + 29.9 f'_{co} + 1053) \times 10^{-6}$ $K_l = \frac{2 E_f t_f}{D}$ $and\ K_l \geq f_{co}'^{1.65}$, $\varepsilon_{h,rup} = k_{\varepsilon.f}\, \varepsilon_{frp}$ $k_{\varepsilon.f} = 0.9 - 2.3\, f'_{co} \times 10^{-3} - 0.75\, E_f \times 10^{-6}$

The main parameters used in the models are:

D **: Concrete diameter (mm)**
ε_{co} **: Unconfined concrete maximum strain**
ε_{cu} **: Axial strain of confined concrete**
ε_{frp} **: Ultimate tensile strain of FRP composite material (manufacturer's value)**
ε_{lu} **: Maximum transverse deformation of confined concrete during failure**
E_c **: Elastic modulus of unconfined concrete /** $E_c = \frac{f'_{co}}{\varepsilon_{co}}$ **(MPa)**
$E_{frp} = E_f$ **: Elastic modulus of composite material (MPa)**
E_j **: Limitation of lateral stiffness /** $E_l = 2\frac{E_{frp}\, t_{frp}}{D}$ **(MPa)**
f'_{cu} **: Ultimate confined concrete strength (MPa)**
$f'_{co} = f'_c$ **: Unconfined concrete strength (MPa)**
t_j **: Fiber thickness of composite material (mm)**
f_{frp} **: Tensile strength of the FRP material (MPa)**
$f_{lu} = f_j$ **: Nominal lateral stress during failure** $f_{lu} = 2\frac{f_{frp}\, t_{frp}}{D}$ **(MPa)**

4. Analytical model evaluation

Using the collected experimental data, the predicted values of the maximum longitudinal strain of the FRP-reinforced concrete were calculated for each examined model. The comparison of the results between the experimental and the analytical solution is of a great interest. Identical results indicate a correct interpretation and description of the experiment from the proposed model. Graphs and statistical indicators, as described below, are used to investigate the convergence or deviation of the results.

4.1 Graphical display of the results

Diagrams correlate the experimental data with those of the strain models. The most accurate models result in values that fall exactly on 45° angle, meaning that the experimental strain values coincide with the analytical. Values below the line of 45° suggest that the analytical equations predict greater ultimate axial strains than the equivalent strains recorded during an experiment, and therefore the models overestimate the effect of confinement. On the contrary, values above the line of 45° indicate that the analytical expressions underestimate the effect of confinement. However, in the context of this study, the results are considered satisfactory when values fall within a region ranging from an axis that passes through the 30° line to an axis that passes through the 60° line, as shown in the outlined area of Fig. 3.

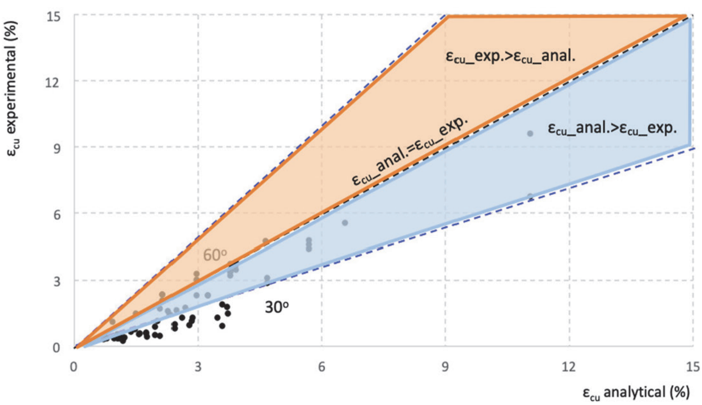

Figure 3. Indicative diagram of experimental vs analytical strain

Fig. 4 shows the correlation of the experimental results to those of the models used. In Fig. 4(a) the Karbhari & Gao [4] model calculates lower ultimate strain than experimentally obtained ones, whereas the Samaan et al. [5] model in Fig. 4(b) much higher. On the contrary, Miyauchi et al. [6], Spoelstra & Monti [7], Ilki et al. [8] and Ozbakkaloglu & Lim [3] seem to be able to make relatively accurate predictions, although the variability is substantial. It should be noted, based on the authors' experience it is very difficult to record the actual average ultimate strains. Especially when strain gauges are used the measured strain describes the point strain and not the average. Thus, it is possible that many reported experimental values are incorrect. This can explain the great variability of the results.

4.2 Statistical analysis

Although the graphs shown in Fig.4 can provide a general idea regarding the ability of the examined models to correctly predict the experimental values of measured strain, it is considered essential to find ways to quantify the results. In order to do so, two statistical indicators were employed: (a) the Root Mean Square Error (RMSE) and (b) the Nash–Sutcliffe model efficiency coefficient (NSE).

RMSE index (Root Mean Square Error)

RMSE index is the standard deviation of residuals, where residuals are the deviations from the regression line [9; 10]. It is expressed by the following equation:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(P_i - O_i\right)^2}{n}} \quad (2)$$

where P_i (Predicted) are the values derived from the analytical equations of the models and O_i (Observed) the results of the experimental tests. Hence, an index value close to 0 denotes that the model describes accurately the experimental data.

The RMSE index is applied to the results of the equations for each model and for specimens of normal strength, confined with all types of fibers. The models are ranked according to the results of the index for each fiber type and then the overall performance (total score) of the model is calculated by adding the ranks, as shown in Table 3. This way, the results are weighted in order to identify the optimal model, i.e. the model that best predicts the ultimate deformation of the confined specimen for each fiber category. In Table 3 the models are ranked in descending order according to the performance of the index by fiber type (e.g. the Miyauchi model has the optimum index value for CFRP confined specimens, therefore it ranked 1st). The ranking for each fiber type is finally added so as to obtain the final classification of the model it the total database. A lower total score indicates the best performing overall model (Spoelstra & Monti [7] in our case, closely followed by Ilki et al.[8] and Ozbakkaloglu and Lim [3]).

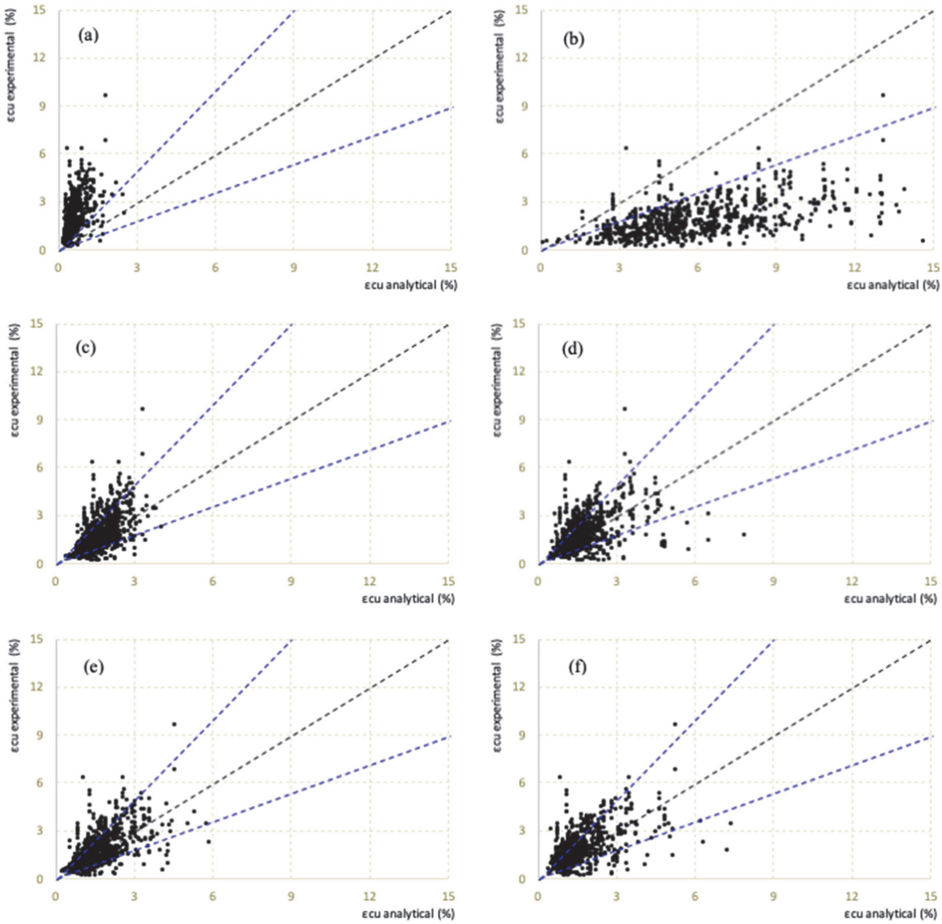

Figure 4. ***e_{cu} experimental vs e_{cu} analytical*** **for CFRP-confined concrete calculated using (a) the Karbhari & Gao model [4], (b) the Samaan et al. model [5], (c) the Miyauchi et al. model [6], (d) the Spoelstra & Monti model [7], (e) the Ilki et al. model [8] and (f) the Ozbakkaloglu & Lim model [3]**

Table 3. RMSE index for normal strength of unconfined concrete

MODEL	RMSE index					FIBER RANKING					TOTAL	FINAL RANKING
	CFRP	GFRP	AFRP	HM_UHM_CFRP	UB_TUBE	CFRP	GFRP	AFRP	HM_UHM_ FRP_	UB_TUBE		
Karbhari & Gao [4]	1.340	1.717	2.115	0.673	1.900	5	5	5	4	5	24	**5**
Samaan [5]	5.141	4.084	3.990	5.885	4.498	6	6	6	6	6	30	**6**
Miyauchi [6]	0.690	1.057	1.438	0.610	1.091	1	2	4	3	3	13	**2**
Spoelstra & Monti[7]	0.817	1.539	1.160	0.334	0.949	3	4	2	1	1	11	**1**
Ilki et al. [8]	0.757	1.051	1.259	0.697	1.081	2	1	3	5	2	13	**2**
Ozbakkaloglu & Lim [3]	0.832	1.260	1.098	0.352	1.126	4	3	1	2	4	14	**4**

4.2.1 NSE index (Nash–Sutcliffe Efficiency)

It is a normalized statistic that determines the relative size of the residual variance compared to the variance of the data. The Nash-Sutcliffe model efficiency coefficient shows whether the graph of experimental versus computational data is described by the line of 1:1 [11; 12]. It is expressed by the following equation:

$$NSE = 1 - \frac{\sum_{i=1}^{n} \left(O_i - P_i\right)^2}{\sum_{i=1}^{n} \left(O_i - \overline{O_i}\right)^2} \tag{3}$$

where P_i (Predicted) are the values derived from the analytical equations of the models and O_i (Observed) the results of the experimental tests. When the NSE value equals to 1 it indicates absolute agreement between the predicted and experimental ultimate strain values, while the value of 0 indicates that the analytical prediction is as accurate as the average of the experimental data.
The NSE index is calculated for each model and the results are detailed in Table 4.

Table 4. NSE index for normal strength of unconfined concrete.

MODEL	NSE index					FIBER RANKING					TOTAL SCORE	FINAL RANKING
	CFRP	GFRP	AFRP	HM_UHM_ CFRP	UB_TUBE	CFRP	GFRP	AFRP	HM_UHM_ CFRP	UB_TUBE		
Karbhari & Gao [4]	-1.134	-1.164	-0.435	-0.984	-2.274	5	5	5	4	5	24	**5**
Samaan [5]	-30.41	-11.24	-4.107	-150.6	-17.35	6	6	6	6	6	30	**6**
Miyauchi [6]	0.435	0.180	0.336	-0.629	-0.079	1	2	4	3	3	13	**2**
Spoelstra & Monti[7]	0.206	-0.739	0.568	0.511	0.182	3	4	2	1	1	11	**1**
Ilki et al. [8]	0.320	0.189	0.491	-1.124	-0.059	2	1	3	5	2	13	**2**
Ozbakkaloglu & Lim [3]	0.177	-0.165	0.613	0.459	-0.150	4	3	1	2	4	14	**4**

According to Table 4 results, each model is more efficient for a certain type of fiber, eg. the Miyauchi et al. [5] model is the most accurate when carbon fibers are used in the composite material. However, since the assessment covers an extensive database with all types of fibers, the Spoelstra & Monti (1999) model appears to have the optimum performance according to the NSE index, as well as in the RMSE index, when calculated for an unconfined concrete specimen of normal strength.

5 Conclusion

This study investigates the performance of a set of analytical models that were developed to predict the maximum strain in compressed confined concrete elements. The performance is analyzed using a quite extensive experimental database containing specimens of compressive strength of 12 to 56 MPa, of different geometry and different fiber confinement. The results of this comparison indicate that:

- Existing analytical models try to predict the maximum strain in confined concrete elements, while taking into account several parameters related to the properties of concrete and confining composite material. The examined analytical models vary in terms of their complexity.
- The unconfined ultimate strain seems to be the most important factor in the performance of a strain model.
- Although some models seem to perform quite well, there is an overall significant variation of the results.
- A relatively simple model such as the one suggested by Spoelstra and Monti [7] performs generally better than some modern complex ones.
- The performance of the models depends upon the fiber type.

References

[1] Bakis, C.E., Bank, L.C., Brown, V., Cosenza, E., Davalos, J., Lesko, J., Machida A., Rizkalla S., Triantafillou, T. (2002). *Fiber-reinforced polymer composites for construction—State-of-the-art review*, Journal of Composites for Construction, 6(2), pp. 73-87.

[2] Papakonstantinou, C.G. (2020): *Fiber Reinforced Polymer (FRP) Confined Circular Columns: Compressive Strength Assessment,* Journal of Engineering Science and Technology Review 13 (6), pp. 1-13.

[3] Ozbakkaloglu, T., Lim, J.C. (2013): *Axial compressive behavior of FRP-confined concrete: Experimental test database and a new design-oriented model.* Composites, pp. 607-634.

[4] Karbhari, V., Gao, Y. (1997): *Composite jacketed concrete under uniaxial compression-verification of simple design equations.* Materials of Civil Engineering, 9(4), pp. 185-193.

[5] Samaan, M., Mirmiran, A., Shahawy, M. (1998): *Model of concrete confined by fiber composites.* ASCE Structural Engineering, 124(9): pp. 1025-1031.

[6] Miyauchi, K., et al. *Strengthening effects with carbon fiber sheet for concrete column.* in *Japan Construction Institute*. 1999.

[7] Spoelstra, M., Monti, G. (1999): *FRP-confined concrete model.* Composites for Constructions, 3(3): pp. 143-150.

[8] Ilki, A., Kumbasar, N., Koc, V. (2002). *Strength and deformability of low strength concrete confined by carbon fiber composite sheets.* Paper presented at the Proc. ASCE 15th Engineering Mechanics Conference.

[9] Wagenmakers, E. J. (2018): *Stevens' Handbook of Experimental Psychology and Cognitive Neuroscience, 4th Edition, Vol.5* (Vol. 3): John Wiley & Sons, NJ USA.

[10] Willmott, C.J., Ackleson, S.G., Davis, R. E., Feddema, J.J., Klink, K.M., Legates, D.R., Rowe, C.M. (1985): Statistics for the evaluation and comparison of models. *Journal of Geophysical Research: Oceans, 90*(C5), pp. 8995-9005.

[11] McCuen, R.H., Knight, Z., Cutter, A.G. (2006): Evaluation of the Nash–Sutcliffe efficiency index. *Journal of Hydrologic Engineering, 11*(6), pp. 597-602.

[12] Nash, J. (1970): River flow forecasting through conceptual models, I: A discussion of principles. *Journal of Hydrology, 10*, pp. 398-409.