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Modal pushover analysis as an alternative to the RSA and NTHA

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Abstract

The modal pushover analysis (MPA) is an improvement of the standard nonlinear performance-based seismic evaluation of structures, based on the first mode constant force distribution over the height. The MPA is based on the theory of dynamics of structures, it retains conceptual simplicity and the computational attractiveness of a standard pushover analysis (SPA). MPA considers higher vibration modes using modal expansion procedure. It can evaluate displacements, inter-storey drift ratios (IDR), and plastic hinge rotations with high accuracy. The total seismic demand is presented as the sum of the effective inertia forces which are in accordance with the invariant force distribution of each significant mode considered. The structure is incrementally pushed to the target displacement, whereby the relationship between force and roof-displacement is successively established, the yield point of the structure estimated, and the capacity curve derived for each mode separately. Target displacement is estimated by solving the governing differential equation of motion or from response spectra. Application of the method is shown on the example of the 10-storey parking garage located at the airport Zürich with dimensions in the plan view of 85 x 105 m approximately.

Key words: modal pushover analysis, nonlinear static analysis, seismic demand, modal expansion, target displacement, inter-storey drift ratio

1 Introduction

The response spectrum analysis (RSA) is currently the most used procedure for the determination of seismic demand and the design of structures; however, it has some known limitations. After numerically combining the modal responses, the physical meaning of the analysis results is difficult to interprete. Furthermore, the RSA procedure is limited to elastic systems. More sophisticated nonlinear time history analysis (NTHA) is seldom used due to the application barriers, such as the necessity of selecting and scaling of ground motion records. Moreover, due to the variety in ground motion contents, the THA should be performed for at least three, usually seven ground motion records to gain reliable results. For the interpretation of the results obtained using NTHA experience and sophisticated software are required. The standard pushover analysis (SPA) is currently the best-established performance-based nonlinear static seismic design procedure based on the invariant first mode force distribution over the height of the structure. Nevertheless, the SPA is limited in that it is not able to account for local damage mechanisms such as inter-storey drift ratio and plastic rotations. As an improvement of the SPA the method, the MPA considers higher vibration modes and can estimate local damage mechanisms with high accuracy.

1.1 Theoretical background

The MPA has been developed by Chopra [4] and prepared for practical use by Chopra and Goel [1-3]. It is derived from the modal analysis of an elastic system in which the coupled displacement vector **u** of the MDF system can be expanded in terms of modal contributions. The basic idea is presented in Figure 1. The governing differential equation of motion is given with Eq. (1).

$$m\ddot{u} + c\dot{u} + ku = p_{eff}(t) \tag{1}$$

$$\boldsymbol{u}(t) = \sum \boldsymbol{u}_{\mathrm{n}}(t) = \sum \varnothing_{\mathrm{n}} \cdot q_n(t) = \sum \Gamma_{\mathrm{n}} \cdot \varnothing_{\mathrm{n}} \cdot D_n(t) \tag{2}$$

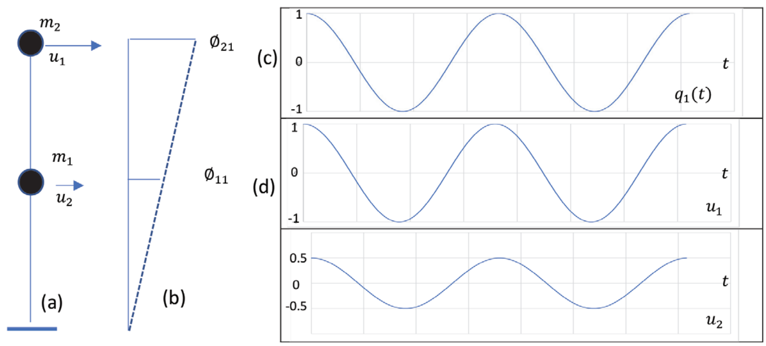


Figure 1. Free vibration in first natural mode a) structure b) vibration mode c) modale coordinate d) displacement u_1 i u_2

$$\boldsymbol{p}_{eff}(t) = -\boldsymbol{m}\iota\ddot{u}_g(t) \quad (3)$$

$$m\iota = \sum_{n=1}^{N} s_n = \sum_{n=1}^{N} s_n \Gamma_n m f_n \quad (4)$$

$$p_{eff}(t) = \sum_{n=1}^{N} p_{eff,n}(t) = \sum_{n=1}^{N} -s_n \ddot{u}_g(t) \quad (5)$$

The modal displacement and force expansion is given with Eq. (2), and Eq. (5), respectively. The modal coordinate $q_n(t)$ can be understood as harmonic excitation with the same period of vibration as the mode investigated. The corresponding displacements $u_{i,n}$, $Ø_n$ is the modal vector, $D_n(t) = \Gamma_n \cdot q_n(t)$ is the target displacement, $\mathbf{s}_n$ is the mass vector as a modal contribution of the n-th mode in the effective modal force $p_{eff,n}(t)$. Finally, ι is the influence vector defined as $\sum \Gamma_n \cdot Ø_n$.
Any response quantity $r_n(t)$ of the structure (member forces, displacements, rotations etc.) can be expressed as modal static response due to the external forces s_n.

$$r_n(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} r_n^{st} \cdot A_n(t) \quad (6)$$

The results of the dynamic analysis of the structure in accordance with Eq. (6) can be obtained performing a few static nonlinear analyses, which is a simple procedure even for nonlinear responses. The entire response of the structure due to the force excitation $p_{eff}(t)$ will be obtained by combining the results of the static analyses, using appropriate combination rule, e.g., SRSS.

1.2 Modal pushover analysis

In MPA the structure is in each mode subjected to the static lateral load distributed over the height of the structure in accordance with the vector s^*_n. The intensity of the load is increased until the peak value of the roof displacement in the given mode u_{rn0} is reached. The force which corresponds to the peak roof displacement u_{rn0} of the building is f_{n0}.

$$s_n^* = m \cdot Ø_n \quad (7)$$

$$u_{rn0} = \Gamma_n \cdot Ø_{rn} \cdot D_n \quad (8)$$

$$f_{n0} = \Gamma_n \cdot m \cdot Ø_n \cdot A_n \quad (9)$$

Both quantities of seismic demand, pseudo acceleration A_n and target displacement D_n are available from response spectra, with the relation between them:

$$A_n = \omega_n^2 \cdot D_n \quad (10)$$

The total response for any quantity required is estimated by combining the peak modal responses of the analysed nonlinear system r_{no},

$$r_0 = \sqrt{\sum_{n=1}^{N} r_{n0}^2} \tag{11}$$

2 Seismic assessment of the parking garage at airport Zürich

2.1 Geometry

The geometry and position of the building, as well as the analytical model, are presented in Figure 2. The structure has been originally built in 1978 as a 14-storey building and 2005 raised by three storeys.

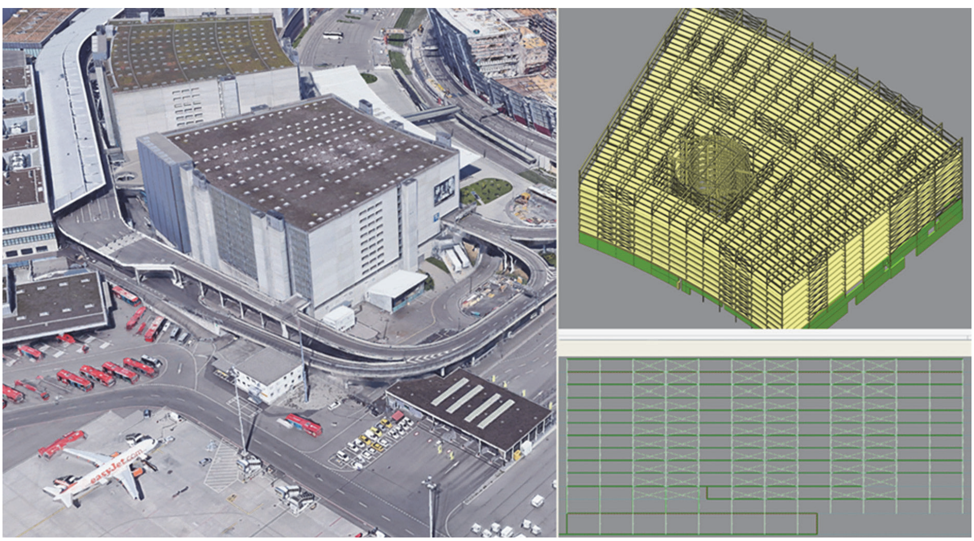


Figure 2. Parking garage at airport Zürich

The structure consists of steel frames braced with members eccentrically attached to the columns (see Figure 3). The structure is symmteric in plan view. Lateral load resisting systems can be reduced to the single braced frame, with the related lateral loads, i.e. masses, the braced frame must take over in case of the earthquake excitation. The floor slabs are prefabricated. Due to the stacking of three additional floors, the weakest bracing members are those situated in the third storey from the top of the structure, consisting of 2[NP 80. The columns are heavy rolled HEB-profiles reinforced with steel plates. Out of 14 storeys, a total of 10 storeys lay above the fixing horizon.

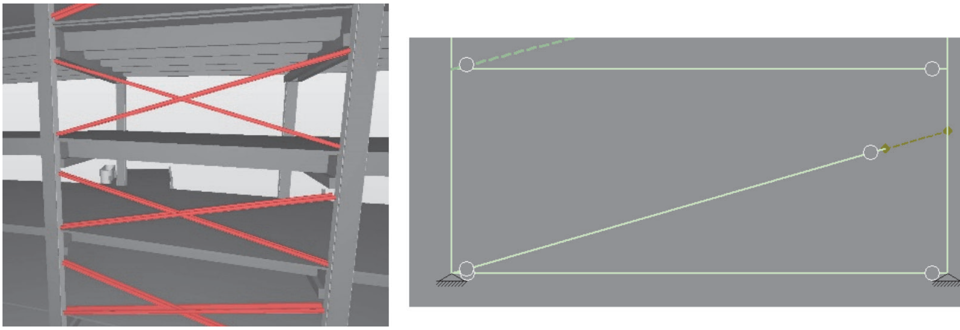


Figure 3. Bracing eccentric connected to the column

2.2 Earthquake excitation

In comparison with seismic-prone regions worldwide, Zürich has low seismic risk with the design ground acceleration of 0.06g. The MPA performed for the actual hazard level has shown that the structure remains elastic in all three vibration modes considered. In order to present the use of MPA in seismic design beyond the yielding point of the bracing, the design acceleration was scaled up to 0.13g.

3 Application of the MPA Procedure

3.1 Natural frequencies and modes of linear-elastic system

The MPA is performed focusing on the first three vibration modes, presented in Figure 4.

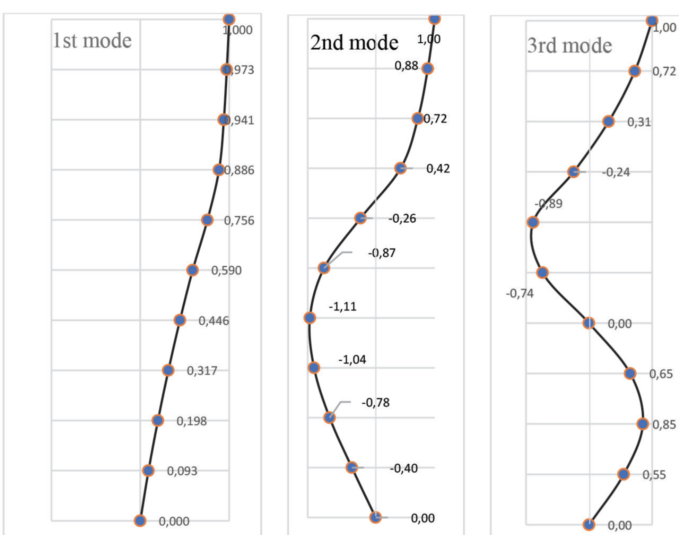


Figure 4. Significant vibration modes of the elastic system

Table 1. Period of vibrations, spectral accelerations, and spectral displacements of the elastic system (left), and the spatial force distribution $s_n^* = m \cdot \emptyset_n$ (right)

	[s]		[m/s²]		[mm]
T_1	3.294	A_1	0.31	D_1	98.8
T_2	1.126	A_2	1.58	D_2	32.4
T_3	0.604	A_3	3.70	D_3	25.5

s^*_1	s^*_2	s^*_3
63,22	63,22	63,22
220,33	199,68	163,93
222,57	169,27	73,23
345,22	164,77	-95,30
241,29	-81,56	-283,31
192,89	-285,58	-241,30
149,28	-371,93	-0,61
100,80	-331,50	207,73
69,29	-271,94	297,37
30,45	-132,03	180,18
0,00	0,00	0,00

The periods of vibration are related to the elastic structure, while the pseudo accelerations and displacements are obtained from the behaviour of the actual inelastic structure (see also Figure 5).

The dynamic properties of the structure are estimated as follows:

$$\Gamma_n = \frac{\sum m_i \cdot \emptyset_i}{\sum m_i \cdot \emptyset_i^2}\,;\ L_n = m\cdot\emptyset_n;\ M_n^* = \Gamma_n \cdot L_n \quad \Gamma_1 = 1.358\,;\ \Gamma_2 = -0.509\,;\ \Gamma_3 = 0.319\,;$$

$$M_1^* = 2220.57t,\ M_2^* = 446.71t,\ M_3^* = 116.45t$$

3.2 Developing of the pushover and capacity curves for mode considered.

The first step in the seismic assessment of the structure is development of the pushover and capacity curves in ADRS format. Using the modal vector force distribution, the modal force is applied to the structure and increased incrementally. The base shear and roof displacement are recorded, and the pushover curve developed, as illustrated in Figure 5 on the left side. The procedure presented can be carried out with any software (The Software Tower 8 has been used here. With the elastic-plastic link element force is limited to the bearing capacity of each bracing member.) for nonlinear structural analysis. The only requirement is that a force limit in accordance with the bearing capacity of a bracing member can be employed in the model. The pushover curves are then transformed into capacity curves as required (see [4]). The bilinear approximation of the modal capacity curves is presented in Figure 5 on the right side. In total, three modes have been considered in this paper covering more than 97 % of the total mass.

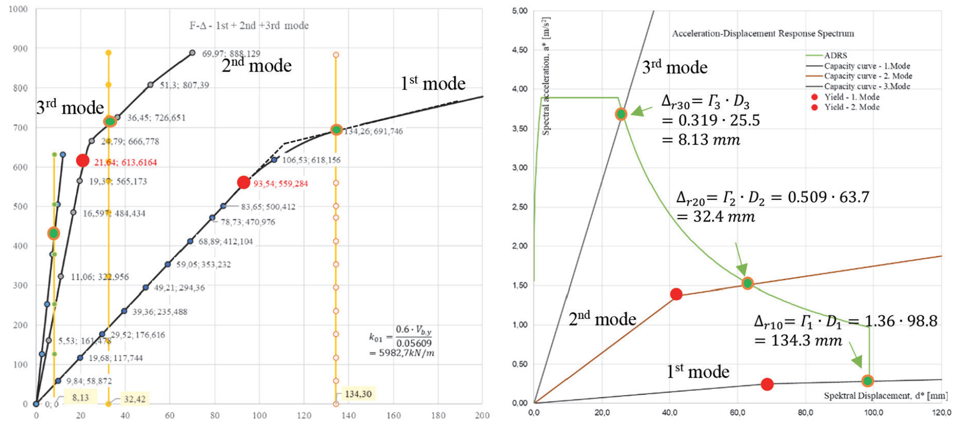


Figure 5. V_b - Δ_{roof} **relationship for the force distribution in accordance with 1st, 2nd, and 3rd mode, left, corresponding representation of the capacity curves in ADRS, right**

3.3 Estimation of the maximum seismic demand of the inelastic system

The peak responses of the equivalent single degree of freedom (SDOF) systems are estimated at the intersection of the inelastic capacity curve and RS. The peak deformations of the SDOF system are D_1 = 98.8 mm, D_2 = 63.7 and D_3 = 25.5 mm, for the 1st, 2nd, and 3rd mode, respectively. The corresponding peak roof displacements are then $\Delta_{r10} = \Gamma_1 \cdot D_1 = 1.36 \cdot 98.8 = 134.3$ mm, $\Delta_{r20} = \Gamma_2 \cdot D_2 = 0.509 \cdot 63.7 = 32.4$ mm, and $\Delta_{r30} = \Gamma_3 \cdot D_3 = 0.319 \cdot 25.5 = 8.13$ mm, whereby the bracings remain elastic in the third mode. The global, inelastic displacement of the structure is governed by the brace yielding ($N_{y2UNP80}$ = 492.4 kN) in 6th and 7th storey in both 1st and 2nd mode respectively, see Figure 6. From Figure 5 is obvious that the displacement capacity of the structure increases the seismic demand for each mode. The complete response of the structure is obtained by combining the modal contributions using appropriate combination rule. It is shown in section 3.4.

3.4 Extraction of the corresponding member forces, displacements, and deformations

The member forces, displacements, and inter-storey drift ratios are estimated for all three significant modes at each stage by reaching target displacement (see Figure 6), and then are combined using SRSS combination rule. The performance-based seismic design focused on the displacements and deformations which are consistent with applied lateral loads, but one can expand the matter of interest to the forces as well. After combining the modal contributions, two different member force results can be expected then: 1) the estimated forces are still in the elastic range, 2) or estimated forces exceed the elastic capacity of the member. The member forces in elements which are intended to remain elastic (columns, for instance) must not exceed elastic capacity after combining by SRSS-rule. For bracings as well, which are intended to develop inelastic deformations, the forces cannot exceed their bearing capacity after modal combination is done. If they do, the force must be recalculated to correspond to the estimated displacements.

3.4.1 The member forces at 6th and 7th storey

Due to the addition of three storeys, the weakest bracing elements in the building are placed at 6th and 7th storey. Applying the SRSS combination rule to the member forces estimated at the target displacement one will obtain for the week members in the 6th and 7th floor:

$$N_{Ed} = \sqrt{492.4^2 + 492.4^2 + 183.53^2} = 719.5 \text{ kN} > N_{Rd} = 492.4 \text{ kN}$$

The forces estimated in this manner exceed the member capacity, which is unrealistic. This is due to the deformations in inelastic range being inherently coupled with their bearing capacity.

The realistic force should be recalculated from real force-deformation relationship. With the strain hardening of approximately 6 % the member force increases in inelastic range (Δ_{pl} = Δ_{tot}-Δ_{el} = 23-14 = 9 mm) for 19 kN to the level of 511.4 kN, being then consistent with the displacement profile estimated "exactly" for the modes considered. The displacement of the structure is governed by bracing elongation, which is estimated realistically and balanced with the force limitation. Some of the iterative analysis required can be performed using commercially available software (Drain -2DX, SAP200 or OpenSees). Despite the behaviour of the structure satisfying all acceptance criteria, from a design point of view, it may be desirable to strengthen the bracings in the 6th and 7th floors, which is a matter of engineer judgment. The comprehensive discussion on this subject can be found in [1].

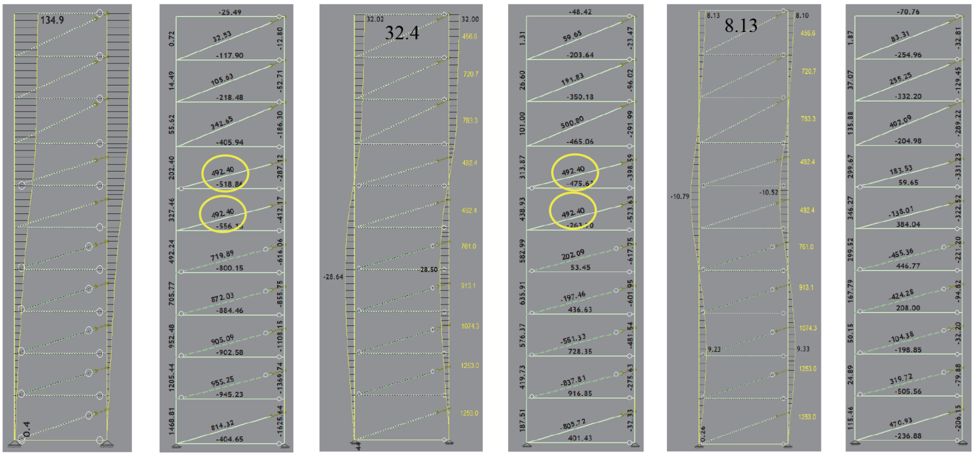


Figure 6. Displacements and member forces according to the 1st, 2nd, and 3rd mode at target displacement

3.4.2 Check of the beam-column connection at the first floor

Using SRSS the axial force in the maximum loaded column, which is intended to remain elastic is: $\sqrt{1369.74^2 + 275.63^2 + 79.9^2} = 1400\text{kN}$. The displacement and bending moment in the node are: $\sqrt{11.2^2 + 10.8^2 + 6.2^2} = 16.74\text{mm}$ and $M_{E,d} = \sqrt{335.96^2 + 357.96^2 + 232.25^2} = 543.1\text{kNm}$ respectively (see also Figure 7).

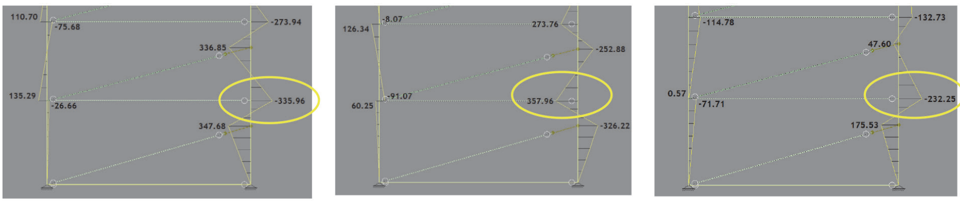
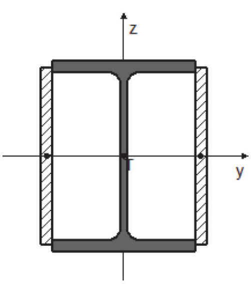


Figure 7.Bending Moments at the basement level for the 1st, 2nd, and 3rd mode

The proof of load carrying capacity and stability of the column on the first floor, which are modelled as linear elastic elements, is carried out here for completeness.



HEB 400 + 2xFL27 x 370

N_d=4398.6 ± 1400= 5798.6 kN; resp. 2998.6 kN

M_d= 543.1+N·Δ= 503.4+5798.6·0.0167= 600.2 kNm

$$\frac{N_{Ed}}{N_{K,Rd}} + \frac{1}{1 - \frac{N_{E,d}}{N_{cr}}} \cdot \omega \cdot \frac{M_{Ed}}{M_{R,d}} \leq 1.0$$

$$N_{cr} = \pi^2 \cdot \frac{EI}{h^2} = 148'189 \text{ kN}$$

$$\frac{\lambda_k}{\lambda_E} = \frac{22.9}{76.4} = 0.3$$

$$N_{k,Rd} = \frac{x_k \cdot f_y \cdot A}{\gamma_{M1}} = \frac{0.96 \cdot 355 \cdot 37560}{1.05 \cdot 10^3} = 12'190.9 \text{ kN; } \omega = 0.6$$

M_{Rd}= 1093+372.67= 1465.66 kNm

$$\frac{5798.6}{12190.9} + \frac{1}{1 - \frac{5798.6}{148189}} \cdot 0.6 \cdot \frac{600.2}{1465.66} = 0.73 < 1 \rightarrow \text{OK}$$

According to SIA263 (2013) § 5.1.9.1 Eq. 49, the proof can be provided.

Even though the focus of the MPA is primary on displacements and deformations, the verification of the forces is shown here for completeness.

4 Comparison of SPA vs MPA

The employment of the MPA aims at covering the contributions of the higher vibration modes to the total response of the structure subjected to the earthquake excitation. In Figure 8 the displacements and IDR for the first mode versus displacements and IDR for three significant modes are presented. However, the IDRs are generally regarded as the cause of the damage in the structure caused by an earthquake. As one can see from Figure 8 the roof displacements, which are the subject of the SPA, are well estimated with the small error of approximately 2.9 %. However, the huge error of approximately 50 %, but small absolute value (merely 5.5 mm) is made on the level of the first floor. The error in IDR is in the range of 54 % on the first and 15-20 % on the 4^{th} to 7^{th} as well as on the 8^{th}, 9^{th}, and 10^{th} level, if only first vibration mode would be considered.

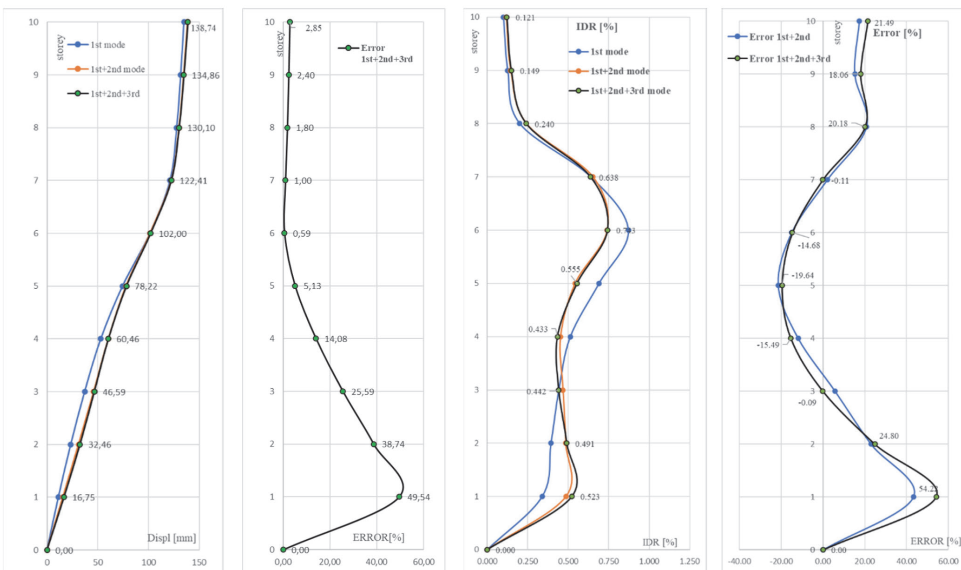


Figure 8. Displacement, IDRs and related Errors produced by considering only first mode

5 Conclusion

The implementation of the MPA has been shown on a real building. The total responses are calculated at target displacements of the first three significant modes, combined by means of an appropriate combination rule and compared with acceptance design criteria. The MPA is easy to use. It si able to account for local damage mechanism such as IDR and plastic hinge rotations with high accuracy.

The IDR, recognized as a significant damage source in the structure, are inherently considered. The method is performance based and governed by inelastic displacement capacity of the structure. The method can be extended to estimate the member forces as well. If the member forces of those elements which are intended to behave in inelastic range, exceed their elastic capacity, the forces must be recalculated using their force-deformation relationship to gain realistic results.

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