

DETERMINATION OF THE DYNAMIC BEHAVIOR OF A BRIDGE STRENGTHENED WITH SHOCK TRANSMISSION UNITS

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Abstract

Shock Transmission Units (STU), also known as Temporary Connection Devices (TCD) or Lock-up Devices (LUD), are mechanical devices that provide a simple and economical way to improve the resistance of existing bridges. They are mainly used for retrofitting existing bridges to accommodate higher-intensity earthquakes and breaking loads defined by new design codes for which the existing bridges do not have sufficient load-bearing capacity. The basic idea of a shock transmission unit is to distribute seismic or other sudden impact loads only to different substructure elements of the bridge so that the bridge behaves as a rigidly connected structure. In the case of slowly acting loads such as temperature, creep, and shrinkage, the shock transmission units are not activated, so that the different parts of the bridge-bearing structure can move independently of each other. They behave like "seatbelts for bridges" because they restrain bridge movement for sudden dynamic loads but allow free movement under slowly acting static loads. To determine the real contribution of STU to the bridge stiffness and consequently, to the global dynamic behavior of the bridge in regular operating conditions, modal parameters such as natural frequency, mode shapes, and damping were determined on the pedestrian bridge, which had been strengthened with shock transmission units. The modal parameters were determined using operational modal analysis and the numerical model. This paper shows the difference between the experimental and numerical modal parameters and draws conclusions about the impact of the shock transmission units on global bridge stiffness. A proposal is also given for the numerical modeling of shock transmission units and their influence on the overall seismic action.

Keywords: shock transmission units, bridges, dynamic parameters, operational modal analysis

1. Introduction

The territory of the Republic of Croatia is in a seismically active area, and in 2020 the country was hit by two major earthquakes. Before that, earthquakes were quite rare, so citizens' awareness of seismic risk was relatively low [1]. Recent events have rapidly increased public interest in seismic risk, and the need to design earthquake-resistant buildings has finally become equally recognized among all participants in the building process. In particular, the need to rehabilitate and strengthen a large number of existing buildings and bridges to withstand the seismic loads prescribed by the existing technical standards (Eurocodes) presented new challenges to all. Such a task became a major challenge for existing bridges since many of them were built a long time ago and for some of them, seismic actions were not considered at all or with much smaller intensity [2–4]. Those bridges should remain in service after the earthquake to accommodate heavy traffic by emergency vehicles and to provide safe passage of lifeline supplies. The above requirements should be met as part of the technical solution during the reconstruction phase. This is hardly achievable with a traditional approach based on strengthening critical bridge parts. Therefore, more advanced methods and systems that can improve the seismic performance of the bridge need to be employed. Based on the functional characteristics of such systems and methods, they can be divided into three types [5], which are shown in Figure 1.

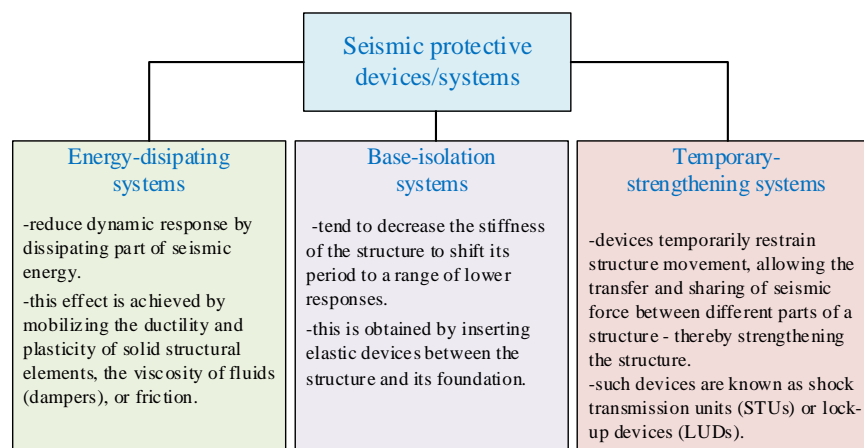


Figure 1. Seismic protective devices/systems

Base-isolation systems are not so suitable solution for existing bridges, since they require demanding and expensive construction procedures to install elastic devices beneath the foundation or other structural elements. On the other hand, the use of energy-dissipating systems and the energy-dissipation approach is quite limited since many of the existing bridges are built on masonry piers, and such substructures do not exhibit ductile behavior. Therefore, temporary strengthening systems also known as Shock Transmission Units (STU) occur as the most applicable ones to be used on existing bridges. They are easily installed and they don't require traffic closure which makes them ideal for investors. In addition, the basic principle of their contribution to overall seismic resistance is relatively easy to understand, making engineers more comfortable to use them in their designs.

The first known use of shock transmission units (STUs) in Europe was in the Netherlands in 1965 on the Oostehshelde Bridge [6]. They were used to transfer traction and braking forces across the central expansion joint. For seismic resistance, they were first used in Italy in 1974 [6]. To the authors' knowledge, shock transmission units are still rarely used in Croatia. A recent example of their use was on the Old Sava Pedestrian Bridge in Zagreb in 2019. Since it took almost fifty years for Croatian engineers to implement the use of shock transmission units in their design concept, the first aim of this article is to introduce the concept of strengthening existing bridges with STU based on the case study example of the Old Sava pedestrian Bridge in Zagreb. The second aim is to investigate the contribution of STUs on overall dynamic behavior through the use of a numerical model updated with experimental data collected through load testing and operational modal analysis.

2. Operational principle of STU

In this chapter, the basic concept of shock transmission units has been explained. To begin with, it is important to distinguish shock transmission units from dampers or energy absorbers. Although they look quite similar, their dynamic behavior and overall impact on the structure is different. STUs have negligible energy absorption capacity due to small piston movement and insignificant hysteresis loops [7]. From the designer's point of view, the STU behaves like an additional support or a rigid link between connected bridge elements, while dampers only increase the damping of an overall structure. In terms of energy handling, the dampers dissipate energy, while STUs distribute energy [8].

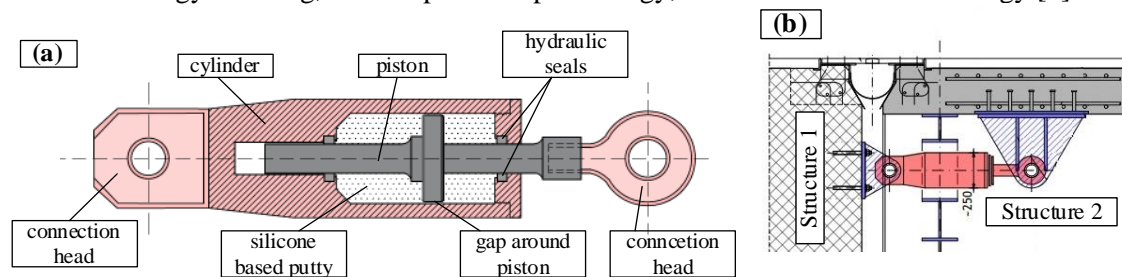


Figure 2. The operational principle of STU. (a) basic parts of the shock transmission unit, (b) the STU unit installed on the case study bridge

The operating principle of STU is based on the fact that rapid passage of viscous fluid through a narrow gap generates considerable resistance, while slow passage produces only minor resistance. As a result, STUs behave as a rigid link transmitting the forces between structural elements for dynamically applied forces such as seismic or braking forces. For slowly applied forces that are mainly caused by temperature, shrinkage, or creep the fluid has enough time to slowly pass the gap allowing free piston movement, therefore only a small amount of force is transmitted between connected elements.

In 1960, the USA Space Exploration Program developed a new material that has a particular thixotropic behavior that makes it optimal for use as a fluid in STU. It is commonly known as a silicone putty (a chemical compound of a boron-filled dimethyl siloxane) and it acts like a rigid body under impact loads, but under slowly applied loads, it deforms easily and without any delay [7]. This makes it ideal for use in shock transmission units as a filler material and it replaced oils and gases that were previously used. This resulted in significant cuts in STUs' need for maintenance which made them more suitable for use on real bridges and structures. Today, STUs are mainly used for (a) multi-span simply supported bridges, (b) multi-span continuous bridges, (c) bridges in seismic areas, and (d) for adjacent continuous viaducts. Although they are mainly used as a unidirectional device for temporarily restraining translational movements, a rotational STU is also produced and it was used on a military pontoon bridge [9].

3. Bridge studied

The Old Sava Bridge in Zagreb, also known as the "Blue Bridge" or "Sava Pedestrian Bridge", was designed by famous Croatian bridge engineer Milivoj Frković and was built in 1939. The new bridge superstructure consisted of steel and concrete bridge superstructure was built on an existing masonry substructure dating back to 1892. Previously, the existing superstructure had been constructed as a system of simply supported steel trusses with wooden deck elements. The design and construction technology of a new span structure was quite ahead of its time. According to the references [10–13], the main advancement was in design and welding technology. At the time, welding technology had difficulties when it came to welding thick steel elements together, but those obstacles have been mastered by the engineers (Fig. 2a) and the thick flanges (more than 90 mm) were successfully welded together, as the later welding tests revealed (Fig. 2b).

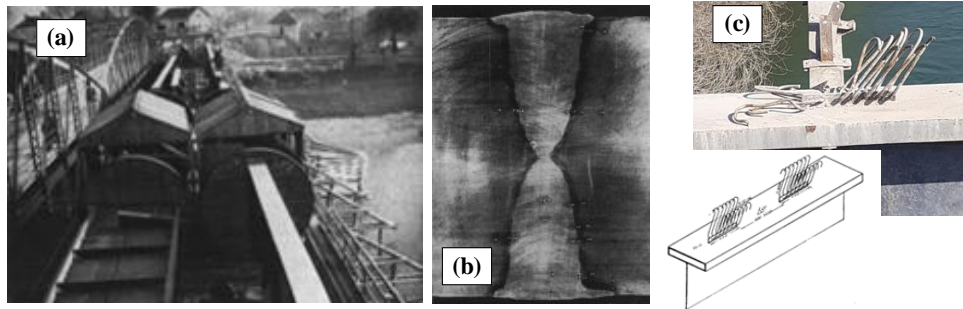


Figure 3. Details from the building process [11]. (a) welding technology, (b) weld quality control, (c) technical detail of the shear connector between the main girder and a concrete slab

Advancement in design is based on the designer's intention to use the reinforced concrete deck as a wind bracing. The installed shear connectors (Fig. 2c) contributed to the bond reaction between the main girders and the bridge deck. The favorable composite response of the superstructure was later demonstrated during the load tests. This encouraged Croatian engineers to further research the field of composite structures in bridge construction. All this led to the recognition of the importance of the bridge for the Croatian cultural heritage and the bridge was placed under strict monument protection. The protection guidelines and the poor condition of the bridge, caused by aging and poor maintenance, were a major challenge for the 2019 bridge reconstruction project. The complete concrete bridge deck had to be removed (fig. 3b) due to the existing damage, so the bridge structure needed to be reconstructed to meet all existing technical standards.

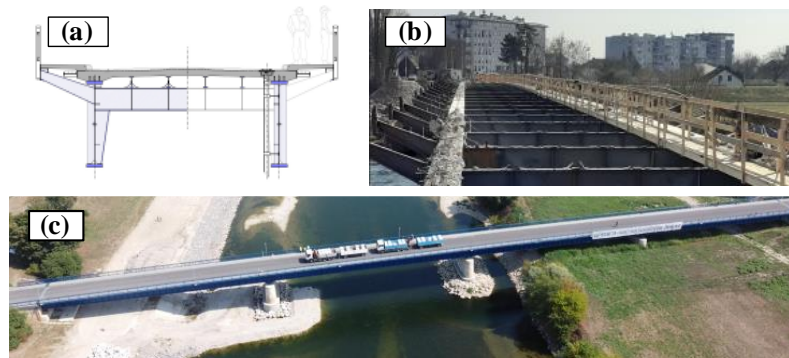


Figure 4. Bridge reconstruction phases. (a) bridge cross-section designed in 2019., (b) bridge superstructure without bridge deck, (c) bridge after reconstruction

The bridge was initially designed for car traffic and its new use was mainly as a pedestrian bridge. With an additional bearing capacity based on a composite behavior, the bridge was able to withstand all vertical loads without major interventions on the steel superstructure. The main problem was the masonry substructure and bearings, especially for seismic analysis. The bearing configuration of the bridge is shown in Figure 4.

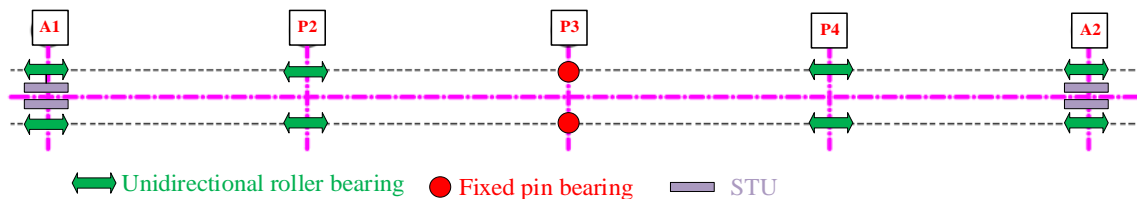


Figure 5. Bridge bearing and STU configuration

The seismic movement in the longitudinal direction is restrained only on the central masonry pier. The shock transmission units were used due to the low bearing capacity and small ductility of the central pier that couldn't resist seismic forces defined according to Eurocode standards. In this way, redistribution of the longitudinal seismic force component to the abutments was ensured, while the free movement was available for regular conditions (creep, shrinkage, and temperature).

3.1. Experimental analysis

According to the Croatian technical standard [14] and the project requirements, load tests had to be performed after the reconstruction of the bridge. The load testing of the bridge was divided into two parts. Static testing was performed according to the Croatian standard for bridge load testing [15]. In this part, the deflections caused by the known static loads have been determined. In the second part, dynamic tests were performed to determine dynamic parameters such as damping, natural frequencies, and mode shapes. The dynamic or modal parameters were determined by operational modal analysis (OMA) [16–19] after the bridge was completed and the STUs have been installed into the bridge. Since the complex mode shapes were expected to affect the limits of the equipment, two sets of measurements were made. First, to capture the horizontal mode shapes and second, to capture vertical ones. For each measurement, a separate experimental model was created using the computer program PULSE LAB SHOP developed by Bruel & Kjaer [20]. Piezoelectric accelerometers (sensitivity of 1000 mV/g) in combination with the Bruel & Kjaer 3560C analyzer have been used to record accelerations at different locations on the bridge and in different directions. For the first measurement setup, named Model_1_HOR, only the data recorded by the accelerometers in the horizontal direction were used for the analysis. Therefore, the mode shapes consisted only of the motions in the horizontal direction. The measurement setup named Model_2_VERT was used with accelerometers oriented only in the vertical direction to obtain vertical and torsional mode shapes.

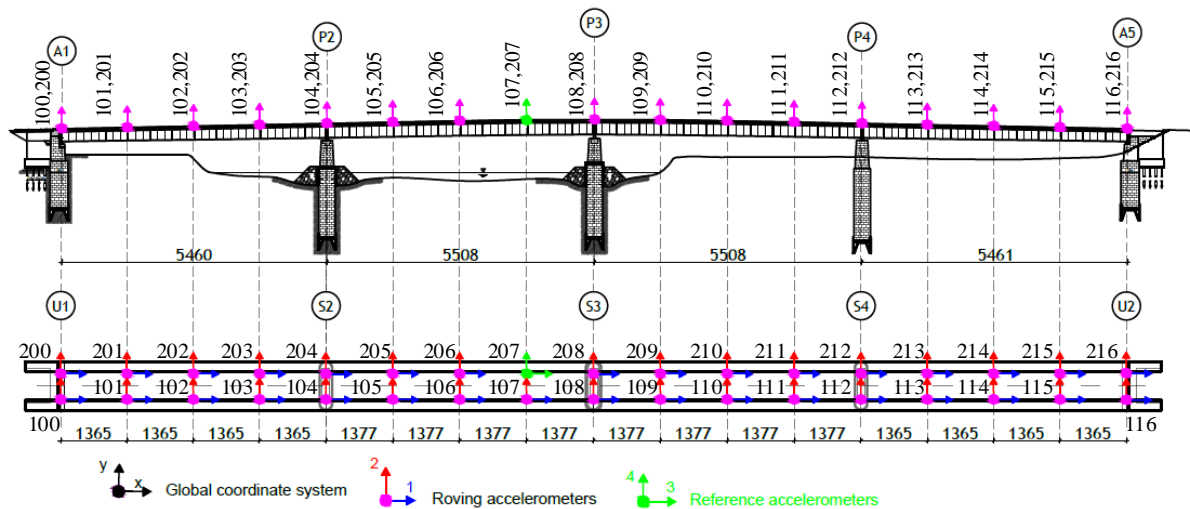


Figure 6. Location of measurement points on the bridge. Red and blue arrows represent the horizontal direction of accelerometers for the first set of measurements while magenta arrows represent the measured direction of accelerometers for the second measurement setup.

For data processing, the OMA techniques developed in the frequency domain, such as **FDD** (**F**requency **D**omain **D**ecomposition method [19]); **EFDD** (**E**nhanced **F**requency **D**omain **D**ecomposition method [21, 22]), **CFDD** (**C**urve-fitting **F**requency **D**omain **D**ecomposition method [23]) were used. The results from each used OMA technique are shown in Figure 7. and obtained results for the studied bridge are shown in Table 1.

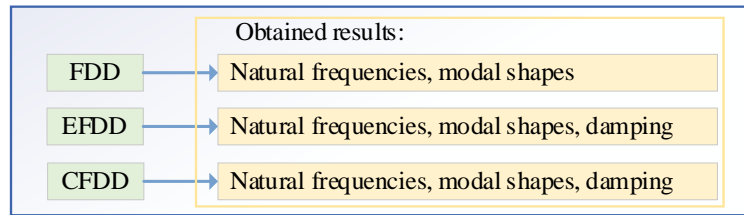
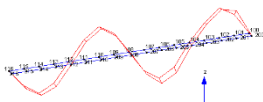
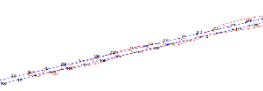
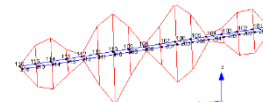
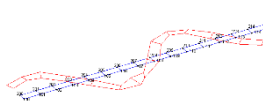
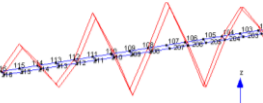
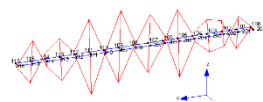


Figure 7. Used OMA data processing techniques and results

Table 1 – Obtained modal parameters

NO	Modal shapes Shape	Natural frequency [Hz]				Mean Value	Damping [%]	OMA model
		FDD	EFDD	CFDD	EFDD			
1.		1,813	1,842	1,859	1,819	2,444	Model_2_VERT	
		1,813	1,797	1,79		1,353	Model_1_HOR	
2.		3,750	3,761	3,792	3,768	1,202	Model_2_VERT	
3.		4,625	4,637	4,636	4,633	1,159	Model_1_HOR	
4.		6,313	6,336	6,320	6,323	1,618	Model_2_VERT	
5.		8,750	8,77	8,760	8,76	1,002	Model_2_VERT	

3.2. Numerical analysis

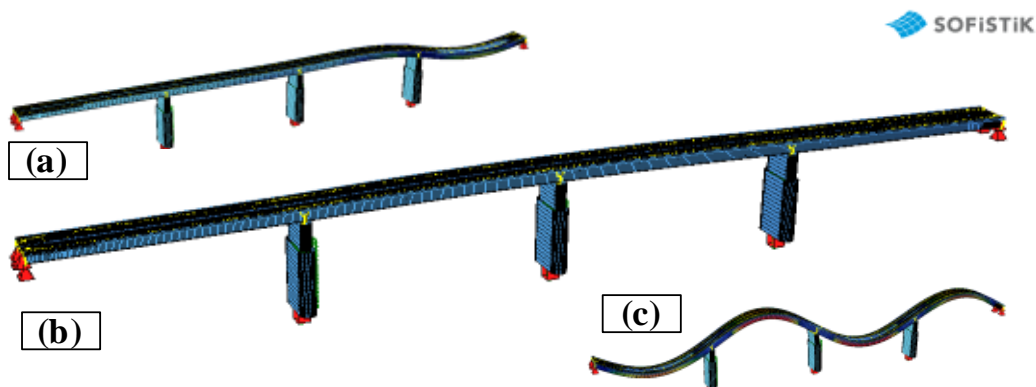


Figure 8. Numerical model of the bridge. (a) The deformed 3D model under the load test at the last span, (b) 3D FEM model, (c) First numerically obtained modal shape at the frequency of 1,798 Hz

Modal analysis and modal parameters are fundamental tools for understanding the dynamic behavior of the structure. In the previous chapter, the modal parameters such as natural frequencies and mode shapes were measured experimentally while the shock transmission units were installed in the bridge. Therefore, the contribution of the STUs could not be determined by experimental tests alone. So, an updated numerical model (Fig. 8) was used to determine the influence of the STUs on the structural dynamic parameters of the bridge.

The initial numerical model was developed in "SOFiSTiK", a commercial software specialized in structural analysis and engineering [24]. The main girders, cross girders, and secondary longitudinal beams were modeled as beam elements while the RC slab was modeled with area finite elements. Bridge piers were also included in the numerical model, along with STUs. STU was modeled as a beam element connecting the bridge RC deck with an additional pinned support at the abutment. The cross-section of this element is assumed to be equal to the cross-section of the STU's piston.

The defined numerical model was updated based on experimental data until a sufficient similarity between the results from the numerical model and those measured on the real structure was achieved. For the updating process, a direct method was used [25]. The method is based on the manual modification of structural parameters such as geometry, material parameters, and boundary conditions.

In Figures 9. and 10. difference between the numerically and experimentally obtained data from the static load test is shown.

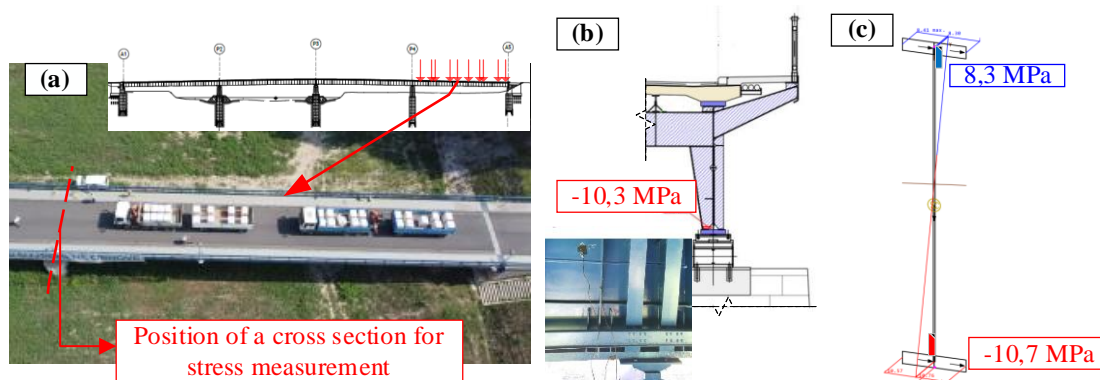


Figure 9. Test load in the last span. (a) Position of test load on the bridge (b) Measured stresses (c) Calculated stresses from an updated numerical model

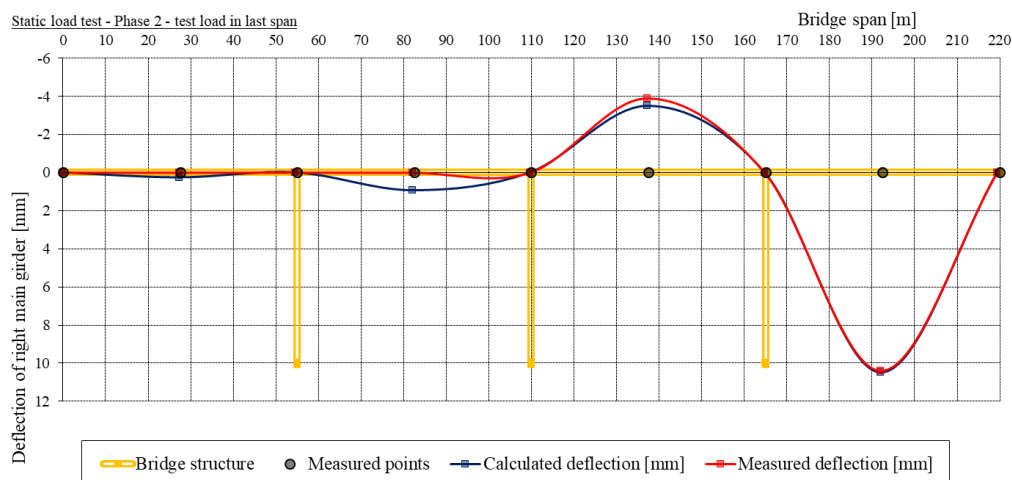

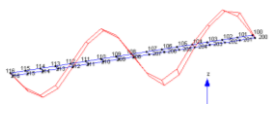


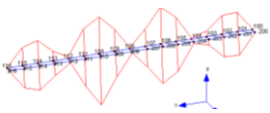


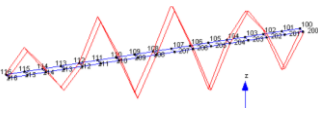


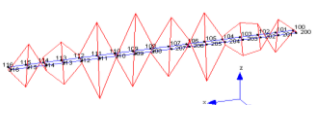
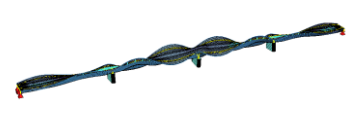



Figure 10. The difference in measured and calculated deflection of the right main girder under the test load in the last span.

The same updated model was then used to perform a modal analysis. In the modal analysis, the Lanczos method [26] was used to calculate the eigenvalues and eigenvectors. The primary load case, defined as a combination of permanent loads (self-weight of the structure and additional weight), was used to define the initial state for the modal analysis. In this way, the contribution of deflection and forces from permanent loads is considered.

Since a satisfactory match between experimental and calculated results for static and dynamic tests has been obtained, it was concluded that an approximation of a real structure with a numerical model was appropriate (at least the flexural stiffness of the superstructure). The modal shapes and frequencies from the operational modal analysis are used as the ground truth for determining the dynamic behavior of the bridge because they were obtained on the bridge with STUs installed while the bridge was in service. To determine the modal parameters of the bridge without STUs, the STUs were neglected in the calculation (they were removed from the model) in the second run of the modal analysis. The overall differences between the modal parameters (natural frequencies and mode shapes) determined by operational modal analysis and by updated numerical models simulating the bridge with and without STUs are shown in Table 2.

Table 2 – Modal shapes and frequencies obtained from experimental and numerical modal analysis

	Experimental results		Numerical results	
	Results from OMA		with STUs	without STUs
Modal shapes and natural frequencies	Horizontal (longitudinal) mode shapes			
				 $f_0=1,175$ Hz
	First vertical mode shapes			
	 $f_1^{\text{exp}}=1,819$ Hz	 $f_1^{\text{STU}}=1,798$ Hz	 $f_1=1,812$ Hz	
	First torsional mode shapes			
	 $f_2^{\text{exp}}=3,768$ Hz	 $f_2^{\text{STU}}=3,795$ Hz	 $f_2=3,795$ Hz	
Second vertical mode shapes				
 $f_3^{\text{exp}}=6,323$ Hz	 $f_3^{\text{STU}}=5,913$ Hz	 $f_3=5,909$ Hz		
Second torsional mode shapes				
 $f_4^{\text{exp}}=8,760$ Hz	 $f_4^{\text{STU}}=8,615$ Hz	 $f_4=8,619$ Hz		

3.3. Discussion

Considering the updated numerical model, it can be concluded from the results presented in Table 2 that despite the installation of STUs, there is no significant increase or decrease in the natural frequencies of the bridge - the difference between the natural frequencies is less than 2%. The main difference can be seen in the mode shapes. The mode shape with the lowest frequency (1,175 Hz) was not detected in the case of the bridge straightened with STU. This mode shape involves a dominant displacement of the bridge superstructure in the longitudinal direction and thus has a significant influence on the dynamic forces acting on the central bearing and the pier (the longitudinal movement of the bridge deck is only restrained by the support in the middle bearings and by the low friction in other the bearings). This mode shape is critical to the design of the center bearing and pier. If we look at the mode shapes derived from the bridge structure with installed STUs (results of the OMA and the numerical model with STUs), the deflection shape at the frequency of 1,175 Hz is not dominant, and consequently, it is not recognized as a mode shape in the numerical model. This can also be seen in Figure 11, which shows the singular values of the spectral density matrices recorded in OMA Model_1_HOR setup. It can be seen that the first significant pick in frequencies is at 1,819 Hz. Therefore, the bridge with the STUs installed has no natural frequency at 1,175 Hz and the corresponding translational shape is avoided by installing the STUs.

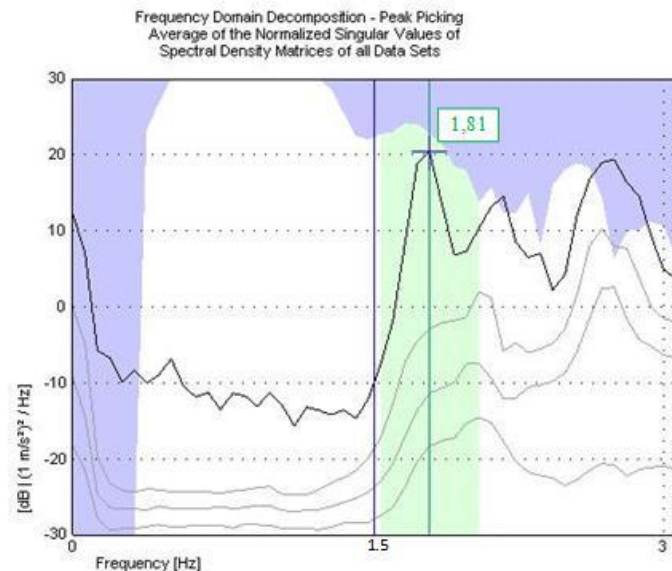


Figure 11. Singular values of spectral density matrices for OMA Model_1_HOR setup.

4. Conclusion

The primary aim of the presented study was to determine the effect of shock transmission units to bridge dynamic behavior. Their contribution was evaluated through modal parameters such as natural frequencies and mode shapes that were obtained from the updated numerical model. An updated numerical model, which performance under the known loads was quite similar to a real bridge, was used for modal analysis in which the contribution of installed STUs has been considered. It is important to note that in this study it was assumed that the STUs are fully activated since they are considered mainly for seismic analysis. The results have shown that the installation of STU makes the bridge stiffer in the directions in which STUs are installed, so the undesirable mode shape is avoided. By incorporating an STU, the first mode shape with a dominant displacement in the longitudinal direction has been avoided, resulting in a much more favorable response to dynamic load such as an earthquake.

While the study included experimental testing, numerical modeling, and updating of the numerical model, some additional conclusions could also be drawn.

The experimental tests and the operational modal analysis have shown that, despite equipment limits, it is possible to obtain horizontal and vertical modal shapes by dividing them into several separate measuring setups in which we combine different measurement directions. For example, as described in chapter 3.1, if we are measuring structures where we expect complex modal shapes (shapes where it is difficult to separate only one or two dominant directions), we can combine different directions to capture the most dominant modal shapes. For continuous bridges, such as the one presented in this study, the authors would recommend measuring the longitudinal direction in combination with the vertical direction in one setup and only the transverse direction for the second setup.

Since numerical modeling and updating a numerical model could be a story for itself, overall, it can be concluded that the data collected through load testing (which is still mandatory in Croatia) has a high value for further use in understanding the behavior of the real structure as it is built and not as it was designed in the project. In this way, a deeper and more reliable understanding of the structural performance of a built structure can be obtained. For this purpose, it is necessary to develop a more advanced and automated method for model updating, since the manual, direct approach is quite time-consuming.

For further studies, it would be interesting to experimentally investigate the process of activation of STUs, by simply conducting experimental modal analysis before and after the installation of STUs on a real structure. Since STUs transfer small amounts of force, even if it is slowly applied, it is important to see if the low-value excitation on which the operational modal analysis is based will activate shock transmission units.

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